Coordination of production and distribution planning

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Abstract: This paper is a computational study to investigate the value of coordinating production and distribution planning. The particular scenario we consider concerns a plant that produces a number of products over time and maintains an inventory of finished goods at the plant. The products are distributed by a fleet of trucks to a number of retail outlets at which the demand for each product is known for every period of a planning horizon. We compare two approaches to managing this operation, one in which the production scheduling and vehicle routing problems are solved separately, and another in which they are coordinated within a single model. The two approaches are applied to 132 distinct test cases with different values of the basic model parameters, which include the length of the planning horizon, the number of products and retail outlets, and the cost of setups, inventory holding and vehicle travel. The reduction in total operating cost from coordination ranged from 3% to 20%. These results indicate the conditions under which companies should consider the organizational changes necessary to support coordination of production and distribution.

Keywords: Production and distribution; Coordination; Planning, Operations management

1. Introduction

Most consumer products flow through a pipeline that begins with production at a plant, followed by transportation to a retail outlet for consumer purchase, perhaps passing through a distribution center on the way. Most companies manage these two functions independently, with little or no coordination between production scheduling and distribution planning. This decoupled approach works acceptably well if there is sufficient finished goods inventory to buffer the production and distribution operations from each other. However, the cost of carrying inventory and the trend to just-in-time operations is creating pressure to reduce inventories in the distribution channel. As a result of this pressure, many companies are exploring closer coordination along the manufacturing/ distribution channel. For example, Johnson & Johnson is testing a linkage between their Band-Aid factory in North Brunswick, New Jersey, and a small retail chain (Fortune, 1990). Procter and Gamble and Walmart have also teamed up to coordinate better the flow of products between them.

It is becoming increasingly clear that companies will need to make the necessary organizational changes that will facilitate coordination of these operational functions and develop an ability to make more complex decisions within this structure. Our work provides an indication as to when such efforts will be most useful and helps in making some critical decisions.

There is a small but growing body of management science research concerned in various ways with the coordination of production and distribution. Glover et al. (1979) developed a network flow model of the production scheduling, inventory and distribution decisions of Agrico Chemical. They embedded this model in a decision support system that was used to analyze both short-run planning decisions and long-range strategic decisions, such as the sizing and location of distribution centers, the size of transportation equipment, and the nature of supply options.

Blumenfeld et al. (1986) considered the problem of scheduling production and distribution for a parts producer supplying parts to a final assembly manufacturer. They considered a very specific scenario that featured one destination per part type, identical production cycles with each production cycle including a production run of every part type, and fixed transportation costs per shipment. They showed that coordination can reduce costs by up to 42% and that maximum savings occur when the demand, item value and variable costs are the same for all items. Blumenfeld et al. (1987) report a successful implementation of this research at GM's Delco electronics division that resulted in a 26% ($2.9 million per year) reduction in logistics costs.

A number of authors have considered an enriched version of the vehicle routing problem in which both the timing and amount of deliveries (as well as the schedule of the vehicle fleet) must be determined so as to ensure that customers do not run out of inventory. Research on this inventory routing problem is reported in Bell et al. (1983), Federgruen and Zipkin (1984), Federgruen et al. (1986), Dror and Ball (1987), and Chien et al. (1988).

Pyke (1987) and Cohen and Lee (1988) studied integrated production/distribution systems under stochastic demand. Pyke developed an analytical model of a simple three node system (factory, finished goods stockpile and single retailer) and examined the properties of the cost functions arising from this model for a single product case. This model does not consider costs or issues related to transportation of goods from the stockpile to the retailer. Cohen and Lee study the impact of various materials management strategies on the cost of production and service levels in an integrated environment. This work integrates the supply chain in terms of inventory distribution without considering their impact on physical distribution strategies.
None of this previous research has considered the integration of production scheduling and vehicle routing, which is the central focus of this paper. Our paper is a computational study to investigate the value of coordinating production and distribution scheduling. The particular scenario we consider concerns a plant that produces a number of products over time. A finished goods inventory of products is maintained at the plant. The products are distributed by a fleet of trucks to a number of retail outlets. The demand for each product at each retail outlet is known for each period of a planning horizon. It is possible to deliver early to a retail outlet, but backorders are not allowed. We assume that the operations of the plant can be modelled as a capacitated lot size problem and that the operation of the fleet of vehicles can be modeled as a standard multiperiod local delivery routing problem.

We compare two approaches to managing this operation, one in which the production scheduling and vehicle routing problems are solved separately and another in which they are coordinated within a single model. This comparison is made for a number of different values of the basic model parameters, which include the length of the planning horizon, the number of products and retail outlets, and the cost of setups, inventory holding and vehicle travel. The reduction in total operating cost from coordination ranged from 3% to 20%. The value of coordination increases as the length of the planning horizon, the number of products and retail outlets, and vehicle capacity increases, as production capacity becomes less binding, and as distribution costs increase relative to production costs.

In the next section, we describe in detail the problem we are considering and present an optimization model of this problem. Section 3 describes our uncoordinated procedure, which includes solution of the scheduling problem using results from Barany et al. (1984), followed by solution of the vehicle routing problem. Our solution procedure for vehicle routing begins with the application of some heuristics, followed by a local improvement procedure that attempts to determine whether changing the timing of a customer delivery can reduce cost. Section 4 describes our coordination heuristic and Section 5 our computational results. A more extensive and earlier description of this work is contained in Chandra (1989).

2. An integrated production and distribution scheduling model

In the problem we consider, several products are produced over time in a single plant. A fleet of vehicles is stationed at the plant to deliver products from the plant to a number of retail outlets. The demand for each product in a period at each retail outlet is known and the problem is to schedule production and distribution so as to minimize the total cost of production setups, transportation and inventory. We now define the parameters, variables and constraints of our model. We assume that [a] denotes the smallest integer not less than *a* and [a] the largest integer not greater than *a*.

Parameters

T = Number of time periods.
m = Number of products.
n = Number of customer locations indexed from 1 to n; 0 represents production facility / depot.
d_{jkt} = Demand for product j at location k in period t .
I_{jk0} = Starting inventory of product j at location k.
P_j = Time for producing one unit of product j.
B = Time units available for production in any given period.
C = Capacity of each delivery vehicle.
S_j = Fixed cost incurred in setting up the facility for product j.
h_{jk} = Inventory holding cost per unit of product j per period at location k.
c_{lk} = Cost of direct travel from location 1 to location k (assume $c_{lk} = c_{kl}$).
v_t = Fixed vehicle cost per route travelled in period t .
The fixed vehicle cost is an allowance for the portion of vehicle depreciation, cost of capital, driver wages, order cost, insurance, etc., which are incurred for each route travelled.

Variables

x_{jt} = Quantity of product j produced in period t .
y_{jt} ={1 if the production facility must be set up for product j in period t; i.e., if $x_{jt} > 0$,
0 otherwise .
q_{jkt} = Quantity of product j delivered to location k in period t .
r_{lkt} = Number of direct trips from location 1 to location k in period t.
I_{jkt} = Inventory of product j at location k in period t.

The Production Scheduling and Distribution problem (PSD) for producing and distributing multiple products from a single facility to several different locations is formulated as

$$\text{Min}\sum_{tj} s_j y_{jt} + \sum_{tjk} h\, I_{jkt} + \sum_{tlk} c_{lk} r_{lkt} + \sum_{lt} v_t r_{l0t}$$

Subject to

$$\sum_{j} p_j\, x_{jt} \le B\ ,\ t=1,\ldots\ldots,T, \qquad (2.1)$$

$$x_{jt} \le M y_{jt} \quad \text{for all } j,\, t, \qquad (2.2)$$

$$I_{j0t} = I_{jot-1} + x_{jt} - \sum_{k=1}^{n} q_{jkt} \quad \text{for all } j,\, t, \qquad (2.3)$$

$$I_{jkt} = I_{jkt-1} + q_{jkt} - d_{jkt} \quad \text{for all } j,\, t;\ \ k=1,\ldots\ldots,n, \qquad (2.4)$$

$$\sum_{\substack{l=0 \\ l\neq k}}^{n} r_{lkt} = \sum_{\substack{l=0 \\ l\neq k}}^{n} r_{klt}, \quad k = 0,\ldots\ldots\ldots\ldots n;\quad t=1,\ldots\ldots T, \qquad (2.5)$$

$$\sum_{l\in s}\ \sum_{k\in s} r_{lkt} \ge \sum_{j}\ \sum_{k\in s} q_{jkt}/C \text{ for all } S \subseteq \{1,\ldots\ldots.,n\}, t=1,\ldots\ldots..,T, \qquad (2.6)$$

$$y_{jt} \in \{0,1\},\quad x_{jt} \ge 0 \qquad \text{for all } jt, \qquad (2.7a)$$
$$q_{jkt} \ge 0,\qquad I_{jkt} \ge 0 \qquad \text{for all } jkt, \qquad (2.7b)$$
$$r_{lkt} \ge 0 \quad \text{and integer for all } lkt \qquad (2.7c)$$

Constraint (2.1) is the capacity restriction on production in every period. Constraint (2.2) requires setups for each product in periods in which production of that product occurs. The parameter M is a sufficiently large positive number which in our computations we take equal to max(B, $\sum_{T=t}^{T}\sum_{k} d_{jkt}$). Constraints (2.3) and (2.4) are the inventory balance constraints (at the production facility and customer locations respectively). Constraint (2.3) requires that distribution be met from inventory and current production. Constraint (2.4) requires that the demand for any item at a customer location be met from amounts held in inventory (at the location) and the quantity that this location receives from the manufacturing facility in that period. Constraints (2.5)-(2.7) define a vehicle routing problem in which we must deliver a quantity $\sum_{j=1}^{m} q_{jkt}$ to customer *k* in period t. Note that we assume multiple deliveries can be made by different vehicles to the same customer. Constraint (2.5) is the trip integrity constraint that ensures that the number of delivery vehicles leaving a location (including the production facility) is same as the number that went to this location. Constraint (2.6) imposes vehicle capacity requirements and insures that the vehicle trip arcs selected form a connected graph.

Constraints (2.5)-(2.6) are essentially multi-stop versions of constraints used for single-stop routing problems. However, it's considerably more challenging to show that these constraints accurately model the multi-stop routing problem. Since once q_{jkt} is fixed, the multi period problem decomposes into T single period problems, we can limit our proof of validity to the single period problem considered in the following theorem.

Theorem. Consider a vehicle routing problem in which q_k units must be delivered to customer k, k = 1, ..., n, by a fleet of vehicles each of capacity c. Let r_{lk} denote the number of direct trips from 1 to k. Then, the values r_{lk} define a feasible solution if and only if they satisfy

$$\sum_{\substack{l=0 \\ l\neq k}}^{n} r_{lk} = \sum_{\substack{l=0 \\ l\neq k}}^{n} r_{kl} \qquad k=0,\ldots\ldots\ldots n \qquad (2.8)$$

$$\sum_{l\in s}\sum_{k\in s} r_{lk} \geq \sum_{k\in s} q_k / C \qquad (2.9)$$

$$r_{lk} \geq 0 \text{ and integer.} \qquad (2.10)$$

Proof. Constraints (2.8) and (2.10) are clearly necessary for feasibility. Values of r_{lk} satisfying (2.8)-(2.10) define a connected graph for which the in degree of each node equals the out degree. Hence, we can trace a set of routes in this graph beginning and ending at the depot and using each arc *(lk)* r_{lk} times. Suppose there are *R* routes. Let S_r denote the set of customers visited on route *r* and R_k the routes containing customer *k.* Let W_{kr} denote the amount delivered to customer *k* on route *r.* These routes comprise a feasible solution if and only if Z * = 0 in the linear program

$$Z^* = \min \sum_{k=1}^{n} \delta_k$$

$$\text{subject to } \sum_{k\in S_r} W_{kr} \leq C, \quad r = 1,\ldots,R,$$

$$\sum_{r\in R_k} w_{kr} + \delta_k \geq q_k, \ k = 1,\ldots,n,$$

$$w_{kr} \geq 0;\ q_k \geq 0,\ k=1,\ldots n.$$

Letting V_r and U_k denote the dual variables for the first and second constraints (shown above) respectively, the dual of this LP is

$$Z^* = \max \sum_{k} q_k u_k - \sum_{r} Cv_r$$

subject to $U_k \leq V_r$, $k \in S_r$, $r = 1, \ldots, R$,

$$0 \leq u_k \leq 1, \quad k = 1, \ldots, n.$$

Since each V_r appears in only one constraint, we can substitute $V_r = \max_{k \in S_r} U_k$ to obtain

$$Z^* = \max \sum_k q_k u_k - C \sum_r \max_{k \in S_r} u_k$$

Subject to $O \leq u_k \leq 1$, $k = 1, \ldots, n$.

We will show that $Z^* > 0$ implies (2.9) is violated, contradicting r_{tk} satisfying (2.8)-(2.10). We first show that if $Z^* > 0$, there is a $\bar{u}$ satisfying $\Sigma_k q_k u_k - C\Sigma_r \max_{k \in sr} \bar{U}_k > 0$ of the form $\bar{U}_k = K, k \in S$, and $\bar{U}_k = 0$, $k \notin S$, for the same K, $0 < K \leq 1$.

Let U^* denote an optimal solution that does not have this property. Let $K = max_k u^*_k$ and let S contain all k with $u^*_k = K$. Let $\bar{R} = \cup_{k \in s} R_k$. If $K\Sigma_{k \in s} q_k - C \mid \bar{R} \mid K > 0$, then $\bar{U}_k = u^*_k$ for $k \in S$, $\bar{U}_k = 0$ for $k \notin S$ provides the required solution. Otherwise, we can reduce the value of u^*_k, $k \in S$, to the next largest u^*_k value without decreasing the objective function. Since there are a finite number of distinct u^*_k values, repeating this argument eventually will produce a solution with equal positive components. Letting S be the set of indices for which $u^*_k > 0$ and $\bar{R} = \cup_{k \in s} R_k$ as before, the fact that the objective function is positive for this solution implies $K\Sigma_{k \in s} q_k - C \mid \bar{R} \mid K > 0$ or $\mid \bar{R} \mid < \Sigma_{k \in s} q_k / C$. Recognizing that $\mid \bar{R} \mid = \Sigma_{l \in \bar{s}} \Sigma_{k \in s} r_{lk}$ for this solution establishes the violation of (2.9).

3. Decoupled production and distribution scheduling

Our decoupled solution of (PSD) is designed to mimic a procedure that we have found common in industry. We first determine a production schedule that minimizes the cost of setups and inventory holding subject to meeting total demand per period. We then schedule vehicle deliveries of products to customers subject to inventory availability as implied by the production schedule.

The production scheduling problem, which we denote by (PS), consists of finding values for X_{jt}' Y_{jt} and I_{jOt} for all jt that minimize $\Sigma_t \Sigma j(_j s_j Y_{jt} + h_{jk} l_{jot})$ subject to (2.1)-(2.3) with $q_{jkt} = d_{jkt}$ for all jkt. Given values for X_{jt}' the distribution scheduling problem, which we denote by (DS), is a multiperiod vehicle routing problem in which we must minimize $\Sigma_t ((\Sigma_k c_{lk} r_{lkt} + v_t r_{lOt})$ subject to (2.3)-(2.7). The algorithms employed for each of these problems are described below in separate subsections.

3.1. *Production scheduling*

Letting $D_{jt} = E_k d_{jkt}$' problem (PS) can be written as

$$\text{Min} \quad \sum_{jt} (s_j y_{jt} + h_{jk} I_{j0t})$$

$$\text{subject to} \quad \sum_j p_j x_{jt} \leq B \quad t = 1, \ldots, T,$$

$$x_{jt} \leq M y_{jt} \quad \text{for all } jt,$$

$$I_{j0t} = I_{j0t-1} + x_{jt} - D_{jt} \quad \text{for all } jt,$$

$$x_{jt} \geq 0, \qquad I_{j0t} \geq 0, \qquad y_{jt} = 0 \text{ or } 1 \quad \text{for all } jt.$$

This is a capacitated lot size problem which we solve to optimality using results from Barany, Van Roy and Wolsey (1984) and Leung, Magnanti and Vachani (1989) to obtain a stronger formulation for the above problem. Pochet and Wolsey (1991) have extended this approach to more general multi-item lot sizing problem. For all j and for any l, $1 \leq l \leq T$, $L = \{1, \ldots, l\}$, $S \subseteq L$, and $L \backslash S = \{j \in L \mid j \notin S\}$, Barany et al. (1984) show that

$$\sum_{t \in L \setminus S} x_{jt} \le \sum_{t \in L \setminus S} \left(\sum_{s=t}^{l} D_{js} \right) y_{jt} + I_{j0l}$$

is a valid inequality and in most cases a facet for the single item incapacitated lotsize problem. Leung et al. (1989) show that (3.1) are facets for the single item capacitated lot size problem and present quantitative evidence on their effectiveness for multi-item problems. In their computational work Barany et al. (1984) also identified a particularly effective member of this family, namely when $S = \{1, ..., l-1\}$ and I is taken to be the first period in which there is no production when the Wagner-Whitin (1958) algorithm is applied to the uncapacitated dynamic lot size problem for product j. In this case, (3.1) reduces to

$$\sum_{t=1}^{l-1} x_{jt} + D_{jl} y_{jl} \ge \sum_{t=1}^{l} D_{jt},$$

which is obviously a valid inequality since it expresses the requirement that we must set up in period *1* if production in the first *1* -1 periods is inadequate to cover demand in the first *1* periods.
To solve (PS), we added (3.2) for all *j* and applied Marsten's (1987) ZOOM/ XMP .

3.2. *Distribution scheduling*

The distribution problem requires scheduling a fleet of vehicles, each of capacity C, that must deliver an amount $\Sigma_j d_{jkt}$ in or before period *t* to each customer *k.* The total amount of product j delivered in any period cannot exceed the amount available as implied by the production schedule determined previously. To solve this multiperiod vehicle routing problem, we first assume that $[\Sigma_j d_{jk\,tt}/C]$ full truck load deliveries are made to customer *k* in period *t.* This leaves $O_{kt} = \Sigma_j d_{jkt} - C[\Sigma_j d_{jk\,t}/C]$ to be delivered as a partial load. We let W_{kt} denote the amount to be delivered to customer *k* in period *t* as a partial load and initially set $W_{kt} = O_{kt}$. This reduces the problem of scheduling partial loads to *T* standard vehicle routing problems which we solve using some well-known heuristics from the literature We then attempt to improve this solution by consolidating the delivery to customer *k* in period *t* with the delivery to customer *k* in an earlier period $t' < t$. Our heuristic terminates once we have made all cost reducing changes of this type that we can find.

The initial vehicle schedule for each day is taken to be the best schedule produced by the following three heuristics Sweep (Gillette and Miller, 1974), nearest neighbour rule (Rosencrantz, Steams and Lewis, 1974), and a feasible insertion rule first described in Chandra (1989) The sweep heuristic is applied *N* times, using each customer as a starting point for the sweep For all of the heuristics, we also applied Lin and Kemighan's (1973) 3-opt interchange heuristic to each route of each schedule created.

We next search for opportunities to improve this starting solution by combining the delivery to customer *k* in period *t* with the delivery to customer *k* in period $t' < t$. The complete local improvement procedure consists of considering potential improvements of this type in order for all customers k= 1,…,n, for all time periods, *t*=2,…,T, and for all *t'*=1,…,t-1.

To evaluate a proposed shift of the delivery W_{kt} we must check feasibility with respect to product availability and compute the change in inventory cost, the reduction in delivery cost in period *t*, and the increase in delivery cost in period *t* ' .A change is feasible if the available supply for each product *j* in period *t'* is sufficient to accommodate the extra product to be delivered (while keeping the production schedule fixed). Recall that the amount q_{jkt} of product j delivered to *k* in period *t* is in general divided between full loads and partial load totalling *Wkt.* In enforcing the inventory availability constraint as we continue to make shifts, we need to specify the product mix of the quantity *Wkt* that's shifted. The maximum feasible load, max *j,* that can be shifted from *t* to *t'* for item *j* is given by

$\max_j = \min\{q_{jkt}, \min I_{jop}\ p = t', t' + 1, \ldots, t-1\}$

Table 1										
Coordination Benefits for Data Set 1										
Prob. No.	*T*	*m*	*n*	*s*	*h*	*c*	*v*	% Cost Decrease from Coordination		
								x	*y*	*z*
1	5	5	25	20	0.25	1	5	3.1	5.9	8.5
2	5	5	25	100	0.50	4	15	3.2	6.0	8.1
3	5	5	25	20	0.75	3	25	3.4	6.3	8.5
4	5	5	25	100	1.00	2	50	2.3	5.5	7.8
5	5	8	50	20	0.50	2	15	5.7	10.0	12.5
6	5	8	50	80	1.00	1	25	4.6	9.4	11.6
7	5	8	50	50	0.75	4	50	6.2	11.6	14.6
8	5	8	50	100	0.25	3	5	3.6	7.5	10.3
9	5	8	50	80	0.50	3	5	5.0	8.4	11.6
10	5	8	50	20	1.00	4	50	5.9	10.1	13.0
11	5	8	50	100	0.75	1	25	4.6	8.6	11.3
12	5	8	50	50	0.25	2	15	6.4	10.8	14.6
13	5	10	25	80	0.75	2	50	5.1	8.9	11.6
14	5	10	25	50	1.00	3	25	4.1	7.5	11.3
15	5	10	25	80	0.25	4	15	5.6	9.1	11.9
16	5	10	25	50	0.50	1	5	4.7	8.3	11.3
17	10	8	25	50	0.50	4	25	6.9	10.9	14.9
18	10	8	25	100	1.00	3	15	5.8	8.5	12.2
19	10	8	25	20	0.75	2	5	6.5	8.9	13.5
20	10	8	25	80	0.25	1	50	7.9	11.7	15.2
21	10	8	25	100	0.50	1	50	6.5	9.2	14.4
22	10	8	25	50	1.00	2	5	4.7	8.9	12.5
23	10	8	25	80	0.75	3	15	6.7	10.0	13.3
24	10	8	25	20	0.25	4	25	9.4	15.9	17.7
25	10	5	50	50	0.25	3	50	8.4	13.4	17.2
26	10	5	50	80	0.50	2	25	6.9	9.4	13.8
27	10	5	50	50	0.75	1	15	8.0	11.5	15.2
28	10	5	50	80	1.00	4	5	6.9	10.3	15.3
29	10	10	50	100	0.75	4	5	7.4	10.6	14.4
30	10	10	50	20	1.00	1	15	10.5	15.2	18.1
31	10	10	50	100	0.25	2	25	9.6	12.8	17.5
32	10	10	50	20	0.50	3	50	12.7	16.9	20.1
	x	= Constrained case (maximum demand = 0.85 * production capacity).								
	y	= Less constrained case (maximum demand = 0.60 * production capacity).								
	z	Unconstrained case (no restriction on production capacity).								

A shift is feasible if and only if $\Sigma_j \max_j \geq W_{kt}$. We then determine the product mix for the partial load that can be feasibly shifted from *t* to *t* ' .Let δ*j* be the amount of item j in the partial load which is being shifted. The δ*j* must satisfy

$$0 \leq \delta j \leq \max_j \ \forall j, \text{ and } \sum_j \delta j = W_{kt}.$$

We obtain values to satisfy these constraints by considering items in order and setting *8j* = min (max*j*, *Wkt*- $\sum_{i=1}^{j-1}$ δ*i)*. When deliveries to *k* are combined the distribution lot sizes at *k* change to *qjkt* = *qjkt* -*8j* and *qjkt'* + δ*j* in periods *t* and *t'* respectively.
The reduction in delivery costs in period *t* as a result of dropping the delivery to customer *k* is given

Table 2
Factorial analysis for Data set 1

T	*m*	*n*	*s*	*h*	*v*	*c*	No. of problems	% Cost Decrease from Coordination		
								x	*y*	*z*
5							16	4.6	8.4	11.2
10							16	7.8	11.5	15.4
	5						8	5.3	8.5	11.8
	8						16	6.0	10.0	13.4
	10						8	7.5	11.2	14.6
		25					16	5.4	8.8	12.1
		50					16	7.0	11.0	14.5
			20				8	7.2	11.2	14.0
			50				8	6.2	10.4	14.0
			80				8	6.1	9.7	13.1
			100				8	5.4	8.6	12.0
				0.25			8	6.8	10.9	14.1
				0.50			8	6.5	9.9	13.4
				0.75			8	6.0	9.5	12.8
				1.00			8	5.6	9.4	12.7
					5		8	5.2	8.6	12.2
					15		8	6.5	10.1	13.3
					25		8	6.2	10.1	13.4
					50		8	6.9	10.9	14.2
						1	8	6.2	10.0	13.2
						2	8	5.9	9.4	13.0
						3	8	6.2	9.8	13.1
						4	8	6.4	10.6	13.8

x = Constrained
y = Less constrained
z = Unconstrained

Table 3a

Detailed costs and CPU times for Data Set 1 (constrained capacity)

Prob.	Uncoordinated			Coordinated			CPU sec
	Setup	Inv	Distrn	Setup	Inv	Distrn	
1	460	30.00	239759.00	500	13.25	232336.08	162.23
2	2400	65.00	871362.10	2500	63.50	843301.13	120.42
3	500	0.00	669233.06	500	52.50	646141.74	156.04
4	2400	85.00	430141.45	2400	63.00	420256.30	103.76
5	800	0.00	1433124.25	800	56.00	1350761.00	312.81
6	3200	0.00	760014.00	3200	39.00	725019.80	337.43
7	2000	0.00	2876528.75	2000	29.25	2696879.31	311.60
8	3700	243.25	2121540.18	3800	188.00	2045828.22	326.57
9	2960	118.50	2139871.65	3200	93.00	2031866.76	346.10
10	800	0.00	2876528.75	800	45.00	2707584.55	329.13
11	3900	46.50	725557.06	4000	42.75	691684.80	318.07
12	1700	246.25	1432052.98	1850	140.50	1339945.98	334.15
13	3680	281.25	895083.00	3760	218.25	849124.84	147.17
14	2250	217.00	1305668.57	2400	179.00	1252053.83	172.28
15	3440	475.50	1746197.10	3840	282.00	1647634.27	164.56
16	2350	138.00	437016.41	2500	75.00	416184.80	142.65
17	3600	362.00	2890332.00	3800	334.50	2689584.93	605.32
18	7400	397.00	2171279.34	7800	266.00	2046965.74	570.18
19	1600	0.00	1441221.13	1600	78.00	1347648.32	684.67
20	5680	671.50	744992.23	5920	753.25	685313.79	708.70
21	7200	701.50	739062.14	7500	669.50	690166.81	666.23
22	3900	88.00	1440003.76	4000	31.00	1372093.15	613.14
23	6000	324.75	2165776.75	6240	278.25	2021138.50	608.33
24	1580	18.00	2888509.02	1600	103.75	2615577.17	711.00
25	2400	79.75	2779021.79	2400	93.50	2544805.61	1316.02
26	3840	136.00	1830186.30	3920	130.00	1703371.69	1299.63
27	2500	0.00	922272.50	2500	67.50	847853.29	1304.93
28	4000	0.00	3652845.84	4000	103.00	3400054.79	1300.55
29	9600	337.50	7040082.23	9700	359.25	6517554.02	1524.00
30	2000	0.00	1785709.25	2000	160.00	1598376.09	1605.19
31	9200	683.75	3681153.75	9600	601.75	3326469.15	1588.72
32	2000	0.00	5363519.03	2000	80.50	4680944.51	1699.06

by $C_{ij} - C_{ik} - C_{kj}$, where *i* and *j* are the customers that precede and follow customer *k* on the route in which it is delivered.

Determining the increase in cost to deliver an additional amount W_{kt} to customer *k* in period *t'* is the most complicated step in evaluating the change in cost for a possible delivery consolidation and requires us to modify heuristically the vehicle schedule in period *t* ' .The change in the schedule is specified below for each of three cases. We let route R denote the vehicle route on which customer *k* is currently delivered in period *t'*.

***Case* 1**. $W_{kt} + W_{kt}\,t' > C$. In this case, we schedule a full truck load delivery to customer *k*. Since $W_{kt} < C$, the remaining load $W_{kt} + W_{kt}' - C < W_{kt}'$ and hence it fits with the other deliveries on route *R*. If this route consists of a single delivery to customer *k,* we attempt to insert this delivery into another route in the least cost position in which it will fit.

***Case* 2.** W_{kt} + W_{kt} ' ≤ C and the additional amount W_{kt} fits on route *R*. In this case, we simply add the delivery of W_{kt} to route *R* and the increase in cost is 0.

***Case* 3**. W_{kt} + $W_{kt'}$ ≤ C but the additional amount W_{kt} does not fit on route *R*. In this case, we add the delivery of W_{kt} to route *R* and split this route into two new routes in a least cost way that results in both new routes being feasible.

In each of the three cases we also apply the three vehicle routing heuristics described earlier in this subsection to the standard vehicle routing problem for period t' and use the result if it is better than the current solution.

Table 3b

Detailed costs and CPU times for Data Set 1 (less constrained capacity)

Prob.	Uncoordinated			Coordinated			CPU sec
	Setup	Inv	Distrn	Setup	Inv	Distrn	
1	320	104.50	239759.00	380	93.00	225659.77	235.00
2	1500	258.00	871362.10	2000	221.50	818544.39	238.16
3	500	0.00	669233.06	500	68.25	627038.60	261.87
4	1500	516.00	430141.45	2000	427.00	405875.36	235.53
5	800	0.00	1433124.25	800	61.00	1289957.61	438.34
6	3120	74.00	725213.34	3200	101.00	656563.21	445.61
7	2000	0.00	2876528.75	2000	46.50	2543436.47	490.22
8	2600	416.50	2120886.41	2500	697.75	1961412.44	439.38
9	2160	666.50	2140691.25	2480	649.00	1960333.26	416.05
10	800	0.00	2876528.75	800	123.00	2585507.81	456.18
11	2700	999.75	725557.06	3000	944.25	662815.25	411.67
12	1300	379.25	1432052.98	1650	360.50	1276305.16	487.08
13	2400	816.00	895083.00	2880	747.75	814812.47	286.23
14	1800	668.00	1305668.57	1950	564.00	1207904.77	273.13
15	2320	284.25	1745906.20	2640	280.00	1585951.45	312.01
16	1550	493.00	437016.41	1750	471.50	400571.60	295.38
17	2250	985.50	2890332.00	2600	981.50	2574587.14	1182.00
18	4500	1971.00	2163952.38	5200	1918.00	1979904.60	1057.08
19	1600	0.00	1441221.13	1600	153.00	1312801.33	1097.17
20	3200	806.75	744992.23	3520	796.75	657124.25	1243.92
21	3900	1495.00	739062.14	4300	1480.00	670187.08	1129.66
22	3450	471.00	1439880.00	3850	431.00	1310444.19	1120.53
23	3600	1558.50	2165215.29	4240	1521.75	1947357.62	1166.05
24	1040	345.25	2888509.02	1180	339.50	2428303.60	1264.11
25	1450	599.50	2770112.00	1700	580.00	2398966.29	1792.27
26	2480	1006.00	1830186.30	3200	972.50	1656584.50	1786.43
27	2400	60.00	922272.50	2450	107.25	815831.01	1796.03
28	4000	0.00	3652845.84	4000	146.00	3277141.77	1797.08
29	6300	2655.00	7040007.28	7400	2646.00	6290316.49	1913.75
30	2000	0.00	1785709.25	2000	228.00	1514106.99	1956.51
31	5300	1764.25	3531917.50	6100	1759.00	3079194.78	1904.50
32	2000	0.00	5363519.03	2000	119.50	4458773.02	2000.16

4. Coordinated production and distribution planning

The procedure we use to coordinate production and distribution planning is a local improvement heuristic for the combined production/ distribution problem (PSD) defined in Section 2. We begin with the production and distribution schedule determined by the procedures outlined in Section 3, and search for cost-reducing changes. The changes we consider are of the same form as those in Section 3.2, that is a consolidation of deliveries to customer *k* in periods t and t', except that we now allow the production schedule to change. We use the procedure outlined in Section 3.2 to evaluate a potential change, except that the feasibility test is modified. In Section 3.2, we required the availability of sufficient product to accommodate the new deliveries in period t'. This inventory availability constraint becomes a production capacity constraint when we allow the production schedule to change. Specifically, we now require that the total amount of product to be prepared in periods 1 through (', including the additional amount W_{kt} cannot exceed t'*B*. Once we find a feasible shift we determine the product mix of the partial load being

Table 3c

Detailed costs and CPU times for Data Set 1 (constrained capacity)

Prob.	Uncoordinated			Coordinated			CPU sec
	Setup	Inv	Distrn	Setup	Inv	Distrn	
1	300	115.75	239759.00	420	115.00	219200.88	373.19
2	1000	538.00	873294.33	1400	518.50	802052.41	368.26
3	500	0.00	669233.06	500	130.50	611974.33	381.55
4	1200	748.00	669007.63	1700	645.00	616544.47	348.90
5	800	0.00	1433124.25	800	131.00	1253465.93	702.06
6	3120	74.00	725213.34	3120	211.00	640653.93	698.00
7	2000	0.00	2876528.75	2000	92.25	2456171.30	735.12
8	1500	945.25	2126992.11	1900	809.25	1907609.01	700.41
9	1920	774.50	287500.56	2320	753.00	253517.47	711.46
10	800	0.00	2876528.75	800	178.00	2502298.01	728.64
11	2400	1161.75	725557.06	3000	1090.50	642419.15	692.08
12	1050	582.00	1426258.12	1300	578.75	1217207.73	709.20
13	1840	1221.75	886840.00	2160	1164.75	782992.44	568.71
14	1750	884.00	1314269.00	1950	841.00	1165565.34	564.00
15	1040	1018.00	1739857.21	1360	996.50	1532619.18	580.22
16	1300	659.50	437016.41	1500	583.50	387419.82	559.19
17	2100	1098.00	2890332.00	2250	1092.50	2458183.47	1689.43
18	4200	2196.00	2165198.75	4600	2190.00	1900738.83	1601.18
19	1600	0.00	1441221.13	1600	253.50	1246042.49	1642.70
20	2320	1176.00	745006.67	2560	1117.75	630902.81	1687.78
21	3000	2004.50	739062.14	3200	2001.00	631496.82	1663.44
22	2850	1042.00	1441135.83	3250	951.00	1260153.85	1625.07
23	3120	1878.00	2163892.90	3440	1875.00	1875764.10	1658.56
24	980	380.00	2890332.00	1180	377.50	2378883.35	1693.85
25	1250	680.25	2770112.00	1350	678.00	2292668.58	2304.02
26	2720	1197.00	1828479.57	2960	1171.50	1575027.86	2250.11
27	2400	60.00	921988.50	2500	205.50	781504.16	2253.67
28	4000	0.00	3652845.84	4000	248.00	3093831.80	2259.31
29	6200	2823.00	7040886.02	6500	2817.00	6028225.10	2296.50
30	2000	0.00	1785709.25	2000	303.00	1461115.79	2354.14
31	4200	2260.00	3531019.00	4400	2255.50	2910704.05	2311.15
32	2000	0.00	5362863.84	2000	272.50	4284253.71	2389.63

shifted using the procedure described in Section 3.2. except that we first allocate δ_j for all those j for which $x_{jt'} > 0$ in the current production schedule. This is done in an attempt to minimize the increase in setup costs due to the shift.
We evaluate the feasibility and change in cost for delivery consolidations for all customers $k = 1,\ldots n$, and all time periods $t = 2, ..., T$, and $t' = 1, ..., t-1$. We recompute an optimal production schedule for the λ feasible changes that yielded the greatest reduction in distribution cost. In our computational work, we used $\lambda = 10$. This number was chosen to tradeoff computational effort against the cost reduction obtained. We found that for many of our test cases, the best reduction in overall cost could be obtained from the 10 best possible combinations that we chose. The production/ distribution schedule with least total cost is taken to be the new current schedule and the process is repeated. We continue until no improving changes can be found.

5. Computational experiments

The procedures described in this paper were applied to 132 distinct test cases to determine the value of production/ distribution coordination as a function of costs and other problem parameters. The test

Table 4

Cost and coordination bebefits for Data Set 2

m	*n*	*C*	Uncoordinated			Coordinated			% cost decrease from coordination
			Setup	Inv	Distrn	Setup	Inv	Distrn	
5	25	8	13750	6000	219466.50	16750	5994	204009.32	5.21
		12	13750	6000	167202.50	17050	5921	151100.47	6.89
		20	13750	6000	98994.18	17200	5835	87443.63	6.96
		40	13750	6000	35012.76	17200	5892	27843.63	6.99
5	50	8	15850	7645	378405.00	18700	7568	350022.37	6.37
		12	15850	7645	318006.08	18850	7485	289260.04	7.59
		20	15850	7645	185727.65	19000	7468	165613.71	8.19
		40	15850	7645	47284.68	19000	7537	38338.93	8.34
8	25	8	19200	6841	322141.81	21750	6695	298895.34	5.99
		12	19200	6841	262499.22	22250	6640	237778.72	7.58
		20	19200	6841	140553.54	22400	6586	123743.69	8.32
		40	19200	6841	43452.35	22400	6599	34648.81	8.41
8	50	8	20150	8627	443647.06	23800	7720	4051979.35	7.56
		12	20150	8627	381063.82	23850	7665	342316.43	8.79
		20	20150	8627	238259.59	25550	7503	209010.32	9.35
		40	20150	8627	67349.46	25600	7503	54864.00	8.49
10	25	8	22500	7900	372285.19	24350	7935	345509.82	6.18
		12	22500	7900	314017.80	24700	7870	284344.31	7.99
		20	22500	7900	194776.00	24850	7792	172694.00	8.81
		40	22500	7900	53003.44	24900	7798	43359.81	8.81
10	50	8	25850	11058	506179.17	27800	10786	460673.22	8.07
		12	25850	11058	440118.02	28250	10612	391644.01	9.75
		20	25850	11058	269854.39	29750	10487	235234.49	10.20
		40	25850	11058	87893.95	29950	10404	72083.00	9.91

cases were organized into three data sets. Data set 1 contained 32 problems and was used to explore the impact of the number of customers and products, the length of the planning horizon and the relative levels of the costs of setup, inventory holding and vehicle travel. Data set 2 contained 24 problems and focused primarily on the impact of vehicle capacity, as well as the number of products and customers. Data set 3 had 12 problems and focused on the impact of the ratio of the fixed cost of a vehicle trip to the cost per mile of vehicle travel.

In all data sets, customer demand for each product in each period was an integer generated independently from a uniform distribution between 0 and 5. Each product had identical processing time per unit and used an identical amount of vehicle capacity per unit, which, without loss of generality, was chosen to equal 1 in each case. The production capacity in data sets 2 and 3 was chosen such that maximum demand over all periods was equal to 60% of production capacity. Data set 1 contained problems with production capacity fixed at three different levels. For the constrained problems, maximum demand was equal to 85% of production capacity. The less constrained problems had maximum demand equal to 60% of production capacity. Finally, some problems had no restriction on production capacity.

To determine the cost of direct travel between customers, we generated coordinate locations for all customers and took the cost of travel to be a constant cost per unit distance (denoted by c throughout this section) times the Euclidean distance between customers. Customer locations were generated in five clusters in a manner intended to capture the characteristics of real problems. First, five North American cities were arbitrarily chosen to represent cluster centers. A sixth city was chosen to represent the manufacturing facility. Next, customer locations were developed around cluster centers using a bi-variate normal distribution. Note that a 25 customer test case comprises 5 clusters of 5 customers each, while 50 customer test case has 10 customers in each of the 5 clusters.

Table 5

Factorial analysis for Data Set 2

M	n	C	Number of Problems	Average % cost decrease from coordination
5			8	7.07
8			8	8.01
10			8	8.72
	25		12	7.35
	50		12	8.52
		8	6	6.56
		12	6	8.03
		20	6	8.64
		40	6	8.49

Customer location was invariate within each of the three data sets. For example, all 25 customer problems and all 50 customer problems in Data set 1 had the same locations, and the 25 customer problem locations were a subset of the 50 customer problem locations.
For each problem in data sets 1 and 3, setup costs, holding costs, and the fixed cost of a vehicle trip did not vary by product, location or time period, i.e., S_j =S, $h_{jk} = h$ and $vt = v$. The values of S, h and v as well as other parameters for Data set 1 are given for each problem in Table 1. For Data set 1, c = 8 for all problems. For Data set 3, for all problems, T = 5, n = 25, m = 5, $ = 100, h = 1 and c = 8. Other parameter values are given in Table 6.

For Data set 2, for all problems T= 5, c = 1 and V_t = 4 for all t. For each customer/product combination, h_{jk} was randomly chosen from a uniform distribution on [5,20] and for each product Sj was randomly chosen from a uniform distribution on [200,1200]. Other parameter values are given in Table 4.

Tables 1-3 provide results for Data set 1, Tables 4 and 5 for Data set 2, and Table 6 for Data set 3. Most of the information in the tables is self-explanatory, but a few comments might be helpful in understanding the tables. For each problem, the total costs of setup (SETUP), inventory holding (INV), and distribution (DISTRN) costs are given for all problems with and without coordination (COORD and UNCOORD). The inventory cost is for all inventory in the system, whether at the plant or customer locations. The distribution cost includes both the fixed costs of vehicle trips and the cost of direct travel between customer locations. The columns labelled 'Percent cost decrease from coordination' or 'Average percent cost decrease from coordination' give the uncoordinated cost minus the coordinated cost divided by the uncoordinated cost and expressed as a percentage.

All CPU times are for the DecStation 2100 and include the time to find the uncoordinated solution followed by the time to execute the local improvement procedure described in Section 4. About half of this time was required to find the uncoordinated solution.

Tables 2, 5 and 6 provide average results for each data set, arranged to highlight the impact of selected problem parameters. From these tables, there are many observations that can be made about how problem parameters impact the value of coordination.
Let us first consider Table 2 concerned with Data set 1. Perhaps the most dramatic effect observable in this table is the significant increase in the value of coordination as the production capacity constraint is relaxed. This is not surprising, since greater production capacity increases the chances that an improvement in the distribution schedule found by the coordination heuristic will be feasible for the production process. In many cases, improvements in the distribution schedule were obtained by consolidating the deliveries to a customer into a full truckload delivery which was made earlier in the planning horizon. This required products to be produced earlier, which tended to be feasible only if the production capacity constraint was relatively loose.

For somewhat similar reasons, the value of coordination increased as the length of the planning horizon and the number of products and customers increased, simply because there were more options available for improving the distribution schedule through consolidation.

Table 6
Sensitivity on *v / c* ratio (results for Data set 3)

Case	*v*	*c*	Uncoordinated			Coordinated			% Change	Average % cost change from coordination
			Setup	Inv	Distrn	Setup	Inv	Distrn		
v / c = 5:										
(a)	5	1	1400	300	233686.90	1500	255	220158.30	5.7	5.65
(b)	10	2	1400	300	467808.50	1500	255	441442.30	5.6	
(c)	15	3	1400	300	698461.20	1500	255	685558.10	5.7	
(d)	20	4	1400	300	935617.10	1500	255	882988.00	5.6	
v / c = 5:										
(a)	10	1	1400	300	234684.70	1600	260	219821.10	6.2	6.18
(b)	20	2	1400	300	469449.30	1600	260	440111.60	6.2	
(c)	30	3	1400	300	702401.50	1600	260	658689.30	6.2	
(d)	40	4	1400	300	938963.20	1600	260	881281.70	6.1	
v / c = 5:										
(a)	25	1	1400	300	236725.10	1600	267	220440.10	6.8	6.83
(b)	50	2	1400	300	474608.10	1600	267	441547.60	6.9	
(c)	75	3	1400	300	711149.80	1600	267	661664.30	6.9	
(d)	100	4	1400	300	949217.10	1600	267	884923.10	6.7	

Since the uncoordinated procedure starts by minimizing setup and holding costs, one would expect this procedure to do well when these costs are dominant. Conversely, coordination should have a high value if distribution costs are high. These hypotheses are supported by the results in Table 2, which show the value of coordination increasing as *v* and c increase, and decreasing as s and *h* increase. Examples of industries with relatively high distribution costs include bottling, packaged goods, milk products and wood products. For example, Grossman's, a wood products firm has a plant in Braintree, MA that shears lumber in various shapes and sizes and distributes them to 80 or more retail outlets in New England area. Their distribution costs are very high as compared to setup costs which consists of changing saw types and making minor adjustments.

Tables 4 and 5 are concerned with Data set 2. Table 4 shows that coordination can lead to reduction in costs for a range of distribution to production cost ratios. Table 5 confirms the observation from Data set 1 that the value of coordination increases as *m* and *n* increase. We can also see in this table that the value of coordination increases as the vehicle capacity C increases. This is because most improvements in the distribution schedule under coordination are obtained by consolidating deliveries into full truckload shipments to one customer or a very small number of customers. If vehicle capacity is small, this pattern tends to occur even without coordination, so the value of coordination is reduced. For example, in the extreme case when C = 1, all deliveries are full truckload shipments to a single customer location so coordination can have no value.

Table 6 is concerned with Data set 3 and shows that the value of coordination increases as *v* / c increases. As this ratio increases, it becomes increasingly economical to travel a greater distance to a larger number of customers in order to construct a full truckload route, so full truckload delivery becomes very economical. Coordination takes advantage of these phenomena by consolidating deliveries in the distribution schedule to create more truckload routes.

In conclusion, we believe our study has shown that, under the right conditions, the value of coordinating production and distribution can be extremely high. The analysis also provides the ability to make the more complex decisions required under coordination. Currently, in most companies, the organization and incentive structure is not designed to support coordination of production and distribution, so that most efforts are directed at improvements *within* these two functions. These efforts are now reaching the point of diminishing returns so the time is right to consider making the organizational changes necessary to achieve coordination between production and distribution. We hope our results provide an indication of when those efforts would be most fruitful.

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