A Dynamic Distribution Model with Warehouse and Customer Replenishment Requirements

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This paper addresses a multiperiod integrated model that plans deliveries to customers based upon inventories (at warehouse and customer locations) and vehicle routes. The model determines replenishment quantities and intervals at the warehouse, and distribution lots and delivery routes at customer locations. We investigate coordination of customer and warehouse replenishment decisions and illustrate their interdependence. Computational experience on randomly generated problems is reported. We show that ordering policy at the warehouse is a function of how goods are distributed to lower echelons and that coordination leads to cost reduction.

Key words: inventory. Logistics, vehicle routeing, heuristics, supply chain coordination

INTRODUCTION

The role of coordination in an integrated manufacturing environment is becoming increasingly critical for the development of efficient strategies towards the management of technology. Needless to say, firms are organized in an integrated fashion even though some do not operate in that way. The benefits of managing operations in an integrated manner are noteworthy: better management of inventory, better response to market changes, and reduction of inefficiencies in individual operations. In this paper we develop an integrated model to determine replenishment policies at a warehouse given that the warehouse is also responsible for devising efficient plans for distributing goods to customers. Such a scenario could exist for most industries -the warehouse could represent an inventory sink at the plant or can be considered as a regional warehouse responsible for meeting consumer demand while being regularly replenished by some manufacturing facility or any other source.

We study the case where a single warehouse distributes goods to spatially distributed customers in order to meet their non-stationary demand (Figure 1). The demand is assumed to be known for the entire planning horizon. Both warehouse and customer locations can hold stock. The warehouse has to determine the size of delivery to each customer in every period and the delivery routes. In order to satisfy the customer demand, the warehouse has to order goods from a higher echelon, say a plant or a supplier. The distribution costs, the inventory costs and the fixed ordering costs (i.e. every time the warehouse places an order for a replenishment stock it incurs a cost) determine how frequently the warehouse should place an order and what would be the size of this order. The ordering policies at the warehouse are affected by the requirement to determine customer loads and delivery routes simultaneously. The size of distribution lot at a customer location depends not only on its own demand but also on the orders of other customers which are visited by the same vehicle. This, in turn, affects the order sizes at the warehouse since the warehouse has to have enough units in stock to meet the distribution lot requirements. Looking at it in another way, the stock at the warehouse restricts the choice of distribution lots and the feasible routes. Consequently, the solution to this integrated problem dynamically determines the capacity requirement of the warehouse.

Past research, which studies ordering policies while considering the impact of distribution, is scarce. While some authors have considered a fixed transportation cost, little research has gone in designing ordering policies for the warehouse which take into account fixed (vehicle usage cost) and variable distribution costs (based on the distance travelled to deliver the demand on any trip). It is quite common in industry to replenish goods to several customers in the same trip by the same

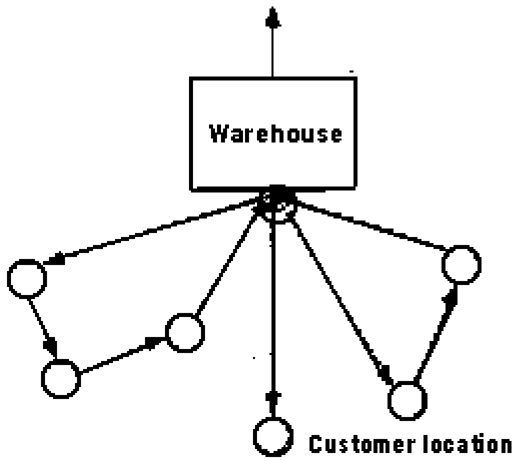

Fig. 1. Warehouse and Customer replenishment structure

delivery vehicle. This leads to the issue of how to consolidate demands of various customers together such that delivery and order costs at the warehouse are minimized. Such an integrated approach towards the logistics function is clearly required in order to enhance service level at reduced costs.

Traditionally, inventory models with multiple customers being replenished from a warehouse have ignored the interaction of these two types of distribution costs. The well-known 'joint replenishment problem' is an example where a fixed cost element is used for each replenishment and a variable cost, if any, is a function of the distribution lot (see Brown[1], GoyaI[2]). The fixed transportation cost assumption (without the consideration of vehicle routes) of previous work does not bring out this interdependence between distribution lots, routes, and ordering frequency. A dominating strategy may be to locate inventory at the customer location in order to gain the advantage of distribution economies of scale (by trying to send full truck loads and by covering several customer demands on a single trip). This may also enhance the service level at customer locations. For example, IBM always locates a small spare parts inventory warehouse next to their mainframe computer locations. Because of such considerations, we include customer's inventory holding cost as a part of the warehouse cost minimization function.

Vehicle routeing problems (VRP), on the other hand, assume that distribution lots are exo- geneously provided. They ignore the inventory considerations at both the warehouse and customer locations. In fact, it is the trade-off between customer replenishment (delivery schedule) and the warehouse replenishment (ordering policy) that determines the best strategy for the entire system.

The literature on integrated models which consider vehicle routeing is quite recent and not extensive. Federgruen and Zipkin[3] and Federgruen *et al.*[4] developed a *single period* model which determined vehicle schedules and delivery sizes simultaneously when customer locations face random demands. Their integrated model minimized holding and shortage costs along with trans- portation costs. The authors adapted the interchange heuristic of the deterministic vehicle routeing problem for solving the integrated problem. Variants of this problem have been addressed by Assad *et al.*[5], Dror *et al.*[6], Dror and Ball[7] and Chien *et al.*[8] Anily and Federgruen[9] and Burns *et al.*[10] addressed the integrated one-warehouse, multi-customer problem under *constant* deterministic demands. Anily and Federgruen[9] derived bounds for the system-wide long-run average (infinite horizon case) costs. They showed that under weak probabilistic conditions these bounds are asymptotically optimal. Burns *et al.*[10] developed solution heuristics to determine where to use direct trips to customers from the warehouse as opposed to delivering the loads to a set of customers on the same trip. In all these cases the warehouse does not hold any inventory.

This paper is organized as follows. In the next section we outline the dynamic replenishment model and discuss the complexity of the problem. In the third section we discuss the approximate algorithm used to solve the integrated problem and in the fourth section we present the results that show the benefits of solving the integrated problem over solving the warehouse replenishment and the customer replenishment problems separately.

DYNAMIC REPLENISHMENT MODEL

We wish to obtain replenishment quantities at the warehouse in conjunction with delivery lot sizes to each customer and delivery schedules that minimize the overall cost. The problem has the following characteristics:

- a finite planning horizon of discrete time periods;
- several products are distributed. Each customer order can comprise more than one type of product for every period;
- demand at each customer location for every period is deterministic;
- vehicles are based at the warehouse and are identical;
- demand can be delivered early but not late.

Define,

[*a*] = smallest integer greater than or equal to *a*, and
[*a*] = largest integer smaller than or equal to *a*.

The parameters and variables that describe the problem are as follows

Parameters

T = number of time periods;
m = number of products;
n = number of customer locations indexed from 1 to n;
O represents the warehouse;
D_{jkt} = demand for product j at location k in period t;
C = capacity of each delivery vehicle.

The relevant costs for determining the optimal production and distribution schedule are:

O_j = fixed cost incurred at the warehouse each time product j is ordered from a higher echelon (this could be plant or a regional warehouse);
h_{jk} = inventory holding cost per unit of product j per period at customer location k (assumed to be equal for both warehouse and customer locations);
C_{1k} = cost of direct travel from location l to location k (assume $C_lk = C_{kl}$); and
v_t = fixed vehicle cost per route travelled in period t.

The fixed vehicle cost includes vehicle depreciation, cost of capital, driver wages, order cost, insurance etc.

Variables

Z_{jkl} = inventory of product j at location k in period t;
q_{jt} = replenishment quantity at the warehouse of product jin period t;
Q_{jkl} = quantity of product j distributed to location k in period t;
w_{kt} = number of deliveries to location k in period t;

$$y_{jt} = \begin{cases} 1 & \text{if the warehouse is replenished with product } j \text{ in period } t\text{, i.e. if } q_{jt} > 0 \\ 0 & \text{otherwise;} \end{cases}$$

$$r_{lkt} = \begin{cases} 1 & \text{if location } k \text{ is visited directly after location } l \text{ in period } t \\ 0 & \text{otherwise.} \end{cases}$$

The Dynamic Distribution problem (DD) with warehouse and customer replenishment requirements can be written as

$$\min\left(\sum_t\sum_l\sum_k c_{lk}\cdot r_{lkt} + \sum_t\sum_j\sum_k h'_{jk}\cdot Z_{jkt} + \sum_t\sum_l v_l\cdot r_{l0t} + \sum_t\sum_j o_j\cdot y_{jt}\right)$$

subject to

$$Z_{j0t} = Z_{j0t-1} + q_{jt} - \sum_{k=1}^{n} Q_{jkt} \qquad \forall j, t \tag{1}$$

$$Z_{jkt} = Z_{jkt-1} + Q_{jkt} - d_{jkt} \qquad k = 1, \ldots, n;\ \forall j, t \tag{2}$$

$$q_{jt} \leqslant y_{jt}\cdot M \qquad \forall j, t \tag{3}$$

$$\sum_{\substack{l=0\\ l\neq k}}^{n} r_{lkt} = w_{kt} \qquad k = 1, \ldots, n;\ \forall t \tag{4a}$$

$$\sum_{\substack{k=0\\ k\neq l}}^{n} r_{lkt} = w_{lt} \qquad l = 1, \ldots, n;\ \forall t \tag{4b}$$

$$\sum_{l\in \bar{S}}\sum_{k\in S} r_{lkt} \geqslant \left\lceil \sum_j\sum_{k\in S} Q_{jkt}/C \right\rceil \qquad \forall S \subseteq \{1, \ldots, n\};\ \forall t \tag{5}$$

$$\begin{aligned} &y_{jt}\in\{0,1\}; && r_{lkt}\in\{0,1\}; && q_{jt}\geqslant 0;\\ &Z_{j0t}\geqslant 0; && Z_{j00}\geqslant 0; && Z_{j0T}=0;\\ &Q_{jkt}\geqslant 0, && Z_{jkt}\geqslant 0; && w_{kt}\geqslant 0 \text{ \& integer } k=1,\ldots,n;\ \forall j,t. \end{aligned} \tag{6}$$

(1) and (2) are the inventory balance constraints at the warehouse and customer locations respectively. Constraint (3) is the replenishment forcing constraint. *M* = a significantly large positive number. Constraints (4a, b) are the well-known assignment restrictions. Constraints (5) are the subtour breaking as well as vehicle capacity constraint. (4) and (5) represent the vehicle routeing component of the problem. (6) lists the non-negativity requirements.

The problem discussed above, DD, is clearly NP-hard. A problem is NP-hard if it has the property that it cannot be solved in polynomial time. Problem DD has two very distinct sub- problems embedded in its structure, each of which is NP-hard. One of the subproblems is the single facility. uncapacitated, multi-product, multi-period warehouse ordering problem (WOP) which seeks to determine the replenishment quantity and the frequency of replenishments at the warehouse. The problem can be expressed as

WOP:

$$\min\left(\sum_t\sum_j o_j y_{jt} + \sum_t\sum_j h'_{j0} Z_{j0t}\right)$$

subject to

Constraints (1), (3), (6).

Florian *et al.*[11] showed that this problem, even with equal demand and zero holding costs, remains NP-hard. Another subproblem is the Distribution Planning Problem (or the customer replenishment problem) which can be written as

$$\min \left(\sum_t \sum_l \sum_k c_{lk} r_{lkt} + \sum_t \sum_l v_t r_{l0t} + \sum_t \sum_j \sum_{\substack{k \\ k \neq 0}} h'_{jk} Z_{jkt} \right)$$

subject to

Constraints (2), (4a), (4b), (5), (6).

This subproblem determines the distribution lot size (i.e. number of units of product j that should be sent to each customer location from the warehouse) and the delivery routes in every period. We assume that the delivery takes place at the end of each period and that there are enough vehicles at the depot (the fixed distribution cast will, however, penalize empty trucks). The unique feature of DP (which differentiate it from a standard vehicle routeing problem) are that the inventory at customer locations needs to be taken into account during distribution and that each customer location can exceed vehicle capacity in any period).
In the next section the solution algorithm used to solve the warehouse and customer replenishment problem is describe and the approximation algorithm for solving the integrated problem is outlined.

APPROXIMATE ALGORITHM

The size and complexity of this problem makes the determination of optimal policies, or even data-dependent bounds on these policies, difficult to obtain in reasonable time. Instead we compare the approximate solution to the overall integrated problem (DD) with the case where the two subproblems (i.e. warehouse replenishment and customer replenishment) are solved separately (i.e. without considering the impact of customer replenishment policies on warehouse replenishment). The difference in two solutions gives us the extent of the reduction in costs due to integration. We intend to illustrate the interdependence of customer and warehouse replenishment policies and the need to consider them together in order to operate an efficient logistics system.

We outline here an iterative approximate algorithm to evaluate the impact of coordinating warehouse and customer replenishment requirements. The idea is quite simple. We start with initial feasible solutions to the warehouse replenishment problem (WOP) and solve the resulting distribution problem (DP) sequentially (call it the 'base case'). Then we see how the warehouse ordering decisions are affected if the delivery schedules for customers (and consequently vehicle routes and loads) are changed. In other words, we estimate the benefits of integration in terms of cost reduction over the case when the warehouse and customer decisions are made independently (i.e. over the 'base case'). We adopt the change (in distribution decisions and/or warehouse decisions) which leads to the greatest reduction in the overall costs (warehouse and customer replenishment). This process is repeated until we reach the stage where there is no further gain by coordinating the two decisions. Figure 2 gives an outline of the approximate algorithm.
There are three main components in this procedure:
(a) Base case.
(b) Problems WOP and DP.
(c) Consolidation process.
We will describe these components first and then explain the approximate algorithm.

Base case
The 'base case' represents the environment where the warehouse and customer decisions are made independently. The steps involved in obtaining a solution to the 'base case' are:

Step 1. Solve the Warehouse Ordering Problem (WOP) with $q_{jt} = E_k d_{jkt}$ to minimize ordering and holding costs.

Step 2. Based on distribution lots obtained form Step 1, the Distribution Planning Problem (DP) with deliveries made on the day the goods are demanded i.e. $Q_{jkt} = d_{jkt.}$

Step 3. Check if any further change in delivery dates (i.e. delivering earlier than demanded) leads to reduction in the distribution costs without changing the warehouse ordering decisions obtained from Step I. If 'yes' then we update the distribution schedule and the distribution costs obtained in Step 2. The Base Case Cost = Cost of Warehouse Ordering + Cost of Distribution.

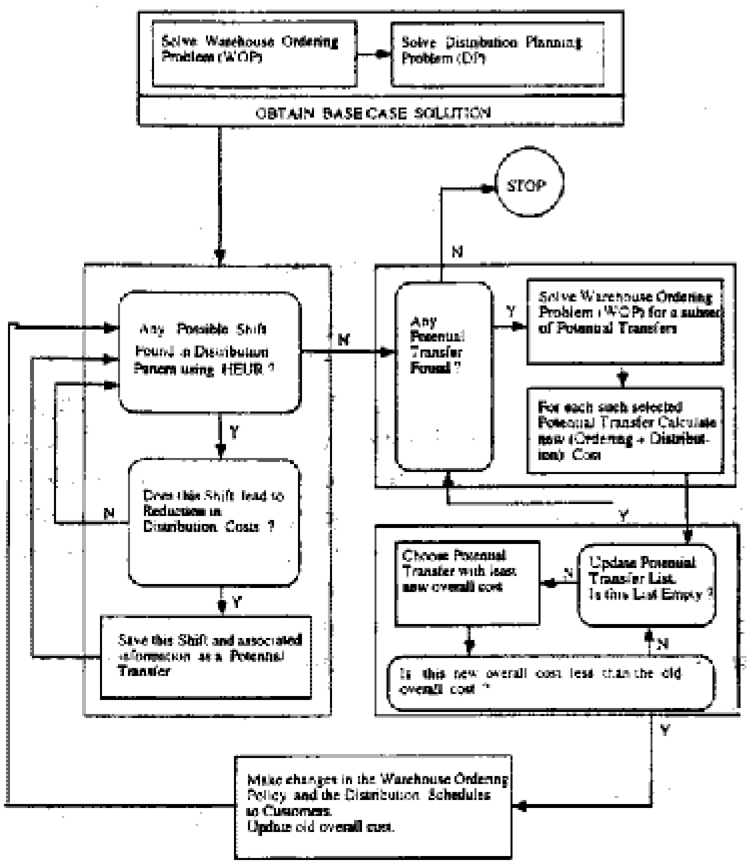

FIG.2. Flow chat for approximate algorithm

In sequentially solving the Problem, as above, there is no consideration of the impact of distribution decisions on warehouse ordering costs and consequently no coordination.

Problems WOP and DP

The warehouse ordering problem (WOP) is solved using the Silver-Meal heuristic[12] for each product in every period. It minimizes the total relevant cost per unit time for the duration of the replenishment quantity. The essence of the heuristic is to find a value for *t* such that the above criterion is met. Then the replenishment quantity that lasts for *t* periods is $Q = \Sigma_{j=1}^{t} d_{ij}$ the demand for product *i* in period *j*. Let the costs associated with this replenishment that lasts for *t* periods be *TC(t)* (= Ordering cost + Holding cost). The heuristic searches for that value of *t* which minimizes this total cost per unit time.

The basic idea is to evaluate *TC(t)/t* for increasing values of *t* (and starting with *t* = 1) until we find a *t* for which *TC(t* + 1*)/t* + 1 > *TC(t)/t.* This gives the number of periods for which the replenishment quantity *Q* should suffice. The period *t* + I represents the period where a new replenishment order should be placed and the process is repeated to find out the value of the order size *(Q)* .This heuristic procedure is found to perform as well as. the more involved Wagner- Whitin algorithm[13] for most realizations of variable demand (Silver and Peterson 14) and is computationally much less taxing. This became an important issue in choosing a procedure for solving WOPsince the approximate algorithm included procedures that were to be solved iteratively and were computationally more intense.

The solution to the distribution planning problem (DP) is obtained by solving heuristically a series of vehicle routeing problems for each period. Since the demand at every location can be greater than vehicle capacity, loads are first decomposed into full and less-than-full truck (or vehicle) loads respectively. At each customer location, the sum of the distribution lot size for all products, i.e. $\Sigma_j Q_{jkt}$ in each period is split into full truck loads, if any, which can be sent directly to the customer from the warehouse and the remainder that could be distributed with similar (less-than-full truck) loads from other customer locations. It is with these 'remainder loads' at each customer location that the vehicle routeing problem is solved in every period. We assume that all items occupy the same volume in the truck. The distribution cost, then, in every period is given by:

Distribution Cost = Direct trip cost + VRP cost + Customer inventory cost

where

Direct trip cost = (k (number of direct trips to each customer k) x (Fixed vehicle cost + $2c_{0k}$))

VRP cost = Vehicle routeing cost + (number of vehicles used x fixed vehicle cost).

The VRP is solved using modifications of TSP heuristics. Savelsberg[15] demonstrated the effectiveness of using a combination of improvement heuristics on a given starting solution to a VRP. We follow a similar approach by first creating several starting solutions (or tours) and then performing tour improvement steps on these starting solutions. We choose the least cost tour as our VRP solution.

We use the following heuristics for generating several starting solutions (or tours) for each period based on the 'remainder loads' at each customer location:
(i) Feasible insertion rule (Chandra and Fisher[16]);
(ii) Nearest neighbourhood rule (Rosencrantz *et al.*[17]); and
(iii) Sweep heuristic (Gillet and Miller[18]).
With the Sweep heuristic we create n schedules, one for each customer as a 'seed point'. In all the above cases, duplicate routes are eliminated. We then apply Lin and Kernighan's[19] 3-opt interchange procedure on each schedule (or set of tours) thus created. The basic idea is to inter- change three arcs, i.e. drop three arcs from the current set of routes and add three new ones if it produces a schedule of lower cost. The process is repeated until there is no further reduction in vehicle routeing costs. Finally, the least cost feasible solution is chosen.

Consolidation process
The process of 'consolidation of deliveries' is used as a vehicle to achieve coordination of customer and warehouse replenishments. Consolidation takes place by shifting delivery of some goods at a customer location from a given period to an earlier period and consequently changing the delivery schedule in both the affected periods.
The consolidation process allows us to determine the impact of moving the delivery date of a customer demand from one period to another on the distribution costs (i.e. vehicle routeing and customer inventory costs). This is achieved by perturbing the distribution schedule using the following 'coordination heuristic' (HEUR):

'Transfer all units of all product types at one customer location from one period to another'.

HEUR takes advantage of distribution economies of scale by trying to utilize full vehicle loads. For example, at customer location k, let Q_{jkt} and $Q_{jkt'}$ -represent the amount of product j being currently distributed in periods t and t' respectively. V sing coordination heuristic, HEUR, we shift distribution of all j at customer location k from t to t'. Hence, the new distribution lot size for all j at location k would be ($Q_{jkt} + Q_{jkt'}$) in period t' and 0 in period t. These shifts may necessitate obtaining new VRP solutions in periods t and t' since the old routes may change due to changes in the makeup of loads at k. Consequently, distribution costs may change.

Chandra and Fisher[16] developed a procedure to estimate the change in distribution costs resulting from such changes, as discussed above, in distribution lots of different product types at a customer location. This estimation procedure negates the need to solve the VRP again for the problem with modified loads in t and t' to obtain new distribution schedules and new distribution costs. We use this procedure of Chandra and Fisher[16] in this research to estimate the change in distribution costs, if any, as a result of shifts from using HEUR.

The 'approximate algorithm' incorporates all the above-mentioned constructs to evaluate the impact of distribution planning on the warehouse ordering policy. The starting point of the approximate algorithm is the determination of the 'base case' solution, i.e. initial warehouse ordering policy and the best distribution plan, each determined independently. It then looks for changes in the delivery schedules arid evaluates their effect on the warehouse ordering policy, i.e. is the existing warehouse ordering policy changed when delivery schedules are changed? If 'yes' then we determine the cost of this change.

The distribution pattern (at the beginning it is the one obtained as part of the 'base case') is perturbed using the coordination heuristic HEUR and the change in the distribution cost, if any, is determined. This process is repeated for all periods $t' < t$ since there is no backlogging allowed. Each time a shift from any t to any t' leads to a reduction in the distribution cost we store that shift and the information associated with it. Then we evaluate the impact of these shifts (that we have saved) which have exhibited reduction in distribution costs by asking the following question: 'Does a particular shift in distribution pattern (which has led to reduction in distribution costs) lead to a reduction in overall warehouse and customer replenishment costs?'

In essence, we first identify all such shifts that lead to a reduction in distribution costs, if implemented. Then, for a subset of such selected shifts we want to know their impact on the warehouse costs. In our experimental study we select ten such shifts which produce the highest reduction in distribution costs. So, for this subset of selected shifts, we solve the modified warehouse ordering problem (WOP) with $qjl = (Q_{jkl} + Q_{jkl'})$ and $q_{jl} = 0$. As a result each selected shift provides a new overall cost (distribution cost + warehouse replenishment cost). Amongst these selected shifts, whichever produces the maximum reduction in overall costs (i.e. new overall cost -current overall cost) is chosen for implementation and the costs are updated. The entire process is repeated by perturbing the new distribution schedule until either there is no potential shift found or when none of the selected shifts yield a new overall cost which is less than the current overall cost. At the beginning of the improvement process the current overall cost is set equal to the 'base cost' and it gets updated with each iteration.

DISCUSSION AND CONCLUSIONS

We describe here an experimental study which examines the effect of coordinating a warehouse's ordering policy and its distribution schedules. The results of this study are used to illustrate the benefits due to coordination and to estimate the impact of structural parameters on the value of coordination. Computations were performed on 33 randomly generated problem sets which are organized under two Data Sets. Data Set I (Table 1(a)) comprises 24 problem sets. This data set is used to study the impact of coordination on the overall costs, order policy at the warehouse (Tables 1 (a, b)) and the role of structural parameters on benefits due to integration (Table 2). Data Set 2 contains 9 problem sets and focuses on the effect of planning horizon on overall costs.

Other parameters for each of the data sets are as follows:

Data Set I: $T = 5$, C = 12, $n = 10$
Data Set 2: C = 14, $n = 10$, $m = 4$, s = 4, $h = 0.25$, c = 1, $v = 5$

In all of the above cases, demand for each product at each customer location was generated independently from a uniform distribution between 0 and 5. Customer locations were developed in a cluster around a centre point using a bi-variate normal distribution ($\sigma = 0.5$ to 1.0). Both coordinates are generated independently. Another location or cluster centre was chosen to represent the warehouse.

As a basis for comparison two different results are reported. The 'Base Case' represents the model where the warehouse ordering and the distribution decisions are made separately and sequentially. Results for the coordination problem are given as 'Integ'. To measure the benefits due to coordination, we report Decrease In cost (= Base Case-Integ/Base Case) as a percentage. The algorithm was programmed in FORTRAN 77 and run on V AX 3100. The total CPU time includes both the sequential and coordinated solution.

Problem #	m	s	h	c	v	% Decrease in cost	CPU sec
1	2	4	0.25	1	5	5.51	22.89
2	2	4	0.25	1	10	6.15	24.63
3	2	4	0.50	1	5	5.45	21.93
4	2	4	0.50	1	10	5.74	23.90
5	2	10	0.25	1	5	5.32	21.88
6	2	10	0.25	1	10	6.63	24.16
7	2	10	0.50	1	5	6.47	22.64
8	2	10	0.50	1	10	5.05	22.36
9	2	25	0.25	1	5	3.78	19.17
10	2	25	0.25	1	10	5.43	23.28
11	2	25	0.50	1	5	3.15	22.38
12	2	25	0.50	1	10	4.31	20.93
13	4	4	0.25	1	5	9.35	61.39
14	4	4	0.25	1	10	9.39	57.71
15	4	4	0.50	1	5	9.95	60.51
16	4	4	0.50	1	10	11.06	59.10
17	4	10	0.25	1	5	10.85	62.64
18	4	10	0.25	1	10	9.58	58.39
19	4	10	0.50	1	5	9.52	58.60
20	4	10	0.50	1	10	8.98	55.09
21	4	25	0.25	1	5	9.52	57.00
22	4	25	0.25	1	10	10.41	60.68
23	4	25	0.50	1	5	8.24	57.53
24	4	25	0.50	1	10	8.92	55.74

Table 1(a) Details of Data Set l

For all data sets, results on each problem set is an average over 25 distinct problems. For example, # I in Table 1 (a) (= 5.51) is the average reduction in cost over 25 problems. Similarly in Table 1 (b), the figures for Warehouse Costs (WHC) and Distribution Costs (DISTRC) and Order Changes (ORCH) are average values obtained over 25 problems.

In Tables 1 (a), l (b) and 2 we present the results of 24 problem sets and summarize the benefits attained by synchronizing ordering policy and distribution plans at a "Warehouse. Table 1 (a) shows the average decrease in cost when the distribution routes and distribution lots are determined along with the determination of order sizes and the timing of order placement at the warehouse. The savings that accrue due to coordination are worth noting. They range from approximately 3 to 11%. Distribution lots, in a way, define the number of units of each product that should be available in stock at the warehouse (as we II as the capacity of the warehouse). This in turn guides the decision as to when the warehouse should place a replenishment order. Consequently, whenever a distribution plan changes leading to a reduction in the overall Cost, the ordering policy at the warehouse gets affected. Table 1 (b) provides evidence of this phenomenon. ORCH represents the average number of changes made to the ordering policy at the warehouse while going from the no coordination (or base case) to the coordination case. Each time an ordering plan or the quantity ordered is altered from the previous solution due to any change in the distribution plan, we call it a change in the ordering policy. ORCH counts a11 such changes in the ordering policy. It is clear that by designing an ordering policy without considering customer distribution schedules we will obtain dominated solutions. The detailed average costs of no-coordination and coordination (for each problem set defined in Table l (a)) are also given in Table 1 (b). WHC is the average warehouse cost while DISTRC is the average distribution cost. As we are considering the uncapacitated case we can easily obtain the actual capacity requirements at the warehouse from the order sizes and the distribution lots.

Table 1(b) Impact of Coordination on overall costs

	BASE CASE		INTEG		
Problem #	WHC	DISTRC	WHC	DISTRC	ORCH
1	39.19	5830.51	35.33	5509.69	7.0
2	39.11	6060.66	34.78	5685.76	7.0
3	39.98	5887.65	35.34	5562.04	8.2
4	40.00	6075.43	35.84	5727.07	6.4
5	79.06	5935.08	73.13	5616.81	5.0
6	78.85	6077.07	71.58	5667.64	5.5
7	95.20	5964.79	85.96	5580.61	5.4
8	93.72	5994.22	87.14	5691.22	5.3
9	143.12	5869.80	138.55	5638.52	3.7
10	140.55	5987.51	134.96	5656.71	3.9
11	176.84	5843.87	169.76	5659.10	3.7
12	179.80	6095.13	173.54	5824.47	3.9
13	78.41	9713.79	68.15	8808.31	5.1
14	78.41	10 136.04	68.91	9183.21	4.7
15	80.00	10 017.05	69.28	9011.68	5.2
16	79.94	10 117.77	69.06	9000.01	5.2
17	156.03	9619.27	139.41	8575.70	5.3
18	156.76	10 203.39	142.60	9224.15	5.2
19	189.40	9746.61	168.22	8809.18	5.1
20	189.60	10 103.46	168.46	9195.44	5.0
21	283.81	9872.44	264.21	8914.97	3.5
22	289.49	10 042.09	266.01	8983.70	3.5
23	359.08	9838.53	335.32	9018.83	2.9
24	360.48	10 134.61	334.78	9215.06	4.3

# of Problems	m	s	h	c/v	% Decrease in cost
12	2				5.25
12	4				9.65
8		4			7.83
8		10			7.80
8		25			6.72
12			0.25		7.66
12			0.50		7.24
12				0.20	7.26
12				0.10	7.68

Table 2 Sensitivity result on structural parameters

Table 2 highlights the impact of various problem parameters on the average percentage decrease in cost and the average number of order changes. The effect of increasing the number of products on distribution is analogous to increasing the number of customers. This helps in consolidating full truck loads which carry loads for a single or few customers and leads to a reduction in costs. Coordination also results in a more efficient utilization of inventory (e.g. by taking advantage of full truck loads) and orders for products are placed closer in time to their distribution. Most of the improvement in overall costs is the result of distribution cost reduction. Consequently, large distribution lot sizes are found to be efficient especially when distribution costs dominate. In almost all cases, inventory at customer location increases. Nevertheless, since we include customer inventory in the warehouse cost minimization function, this causal phenomenon is not alarming.

Similar results are proyided on how ordering policy at the warehouse is affected by various problem parameters. Planning horizon has a significant impact on the order policy and the overall cost at the warehouse (Table 3). As the planning horizon increases, reduction in cost due to coordination increases. This is true since there are more opportunities to reduce distribution costs by shifting the distribution to earlier periods and forming full truck loads. Order policy is found to change as well. However, a longer planning horizon also increases the risk that demand forecast will be less reliable. Hence, prudence is required when choosing a planning horizon.

This paper is an effort to exhibit the advantages of managing the supply chain effectively by considering the interaction of the decisions involved. It provides a theoretical rationale for a warehouse or inventory manager to look at the impact of one set of operational decisions on others. As can be seen from the results, ordering policy at the warehouse is a function of how goods are distributed to lower echelons. When ordering policies are considered in isolation, as is often the case in inventory literature and practice, not only does it result in higher cost of operation but also leads to ineffective management of inventory. It can be seen that small quantities of inventory can bring about a sizeable reduction in distribution costs. Future research includes consideration of integrated models that capture stochastic demand within this framework as well as further exploration of cost characteristics of various types of firms where integration could be beneficial.

Table 3 Impact of planning horizon on ordering policy and overall costs

		BASE CASE		INTEG				
Problem #	T	WHC	DISTRC	WHC	DISTRC	% Decrease in cost	ORCH	CPU sec
1	2	31.63	3626.95	28.15	3313.79	8.25	3.2	7.28
2	3	47.29	5375.26	39.98	4687.69	12.54	3.2	21.93
3	4	62.99	7008.28	54.62	6214.50	11.10	4.6	34.67
4	5	78.20	8958.87	67.08	7883.51	11.95	5.6	63.58
5	6	93.87	10 531.53	80.04	9210.78	12.42	7.0	98.48
6	7	110.16	12 493.97	94.58	10 959.26	12.29	7.7	107.12
7	8	124.93	14 273.05	107.69	12 392.22	13.05	9.0	139.12
8	9	140.82	15 949.69	119.64	13 802.37	13.39	10.9	170.04
9	10	155.67	17 930.47	132.60	15 541.58	13.22	12.1	191.57

Acknowledgements- This research was partially supported by NSERC-Canada grant OGP004Z150. The author wishes to thank Masoud R. Hariri for performing the computational tests.

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