



Models for multi-plant coordination

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Abstract: This paper addresses coordination of production planning among multiple plants in a vertically integrated firm. Currently there is no unified body of literature that deals with this issue. We identify important issues that managers and researchers must address so that production and inventory decisions can be determined for all plants in a manner optimal for the organization as a whole.

Keywords: Multi-plant; Coordination; Production planning

I. Introduction

The threat of foreign competition in an increasingly global marketplace has forced firms to consider ways of improving their manufacturing practices. Attempts in this direction have focused, amongst others, on improvements on the following performance measures: (i) levels of inventory and work-in-process (WIP); (ii) flexibility and responsiveness of the production process through reduced manufacturing lead times; (iii) product quality in terms of number of parts scrapped. The focus on the above issues is not coincidental. Severe competition has forced firms to continuously introduce improved and new products and this has led to considerably shorter product life cycles. In future, therefore, firms will continue to devote considerable resources and time towards choosing strategies which provide good performance on the above measures of inventory, lead time, quality of products and flexibility of production process. An important question that arises in choosing strategies that yield good performance is, how decision making at different levels in the organization should be coordinated so that operating policies are optimal for the organization as a whole. This is critical because without coordination, improvements at one level may be lost due to inefficiencies at another level e.g., reducing inventory at one, stage of the production process will not yield benefits if it accumulates at any succeeding stage.

We distinguish between two broad levels of coordination. At the most general level coordination can be seen in terms of integrating decisions of different functions e.g., facility location, inventory planning and production planning, distribution, marketing etc. We refer to research efforts at this level of functional integration as 'general coordination'. At another level, the problem of coordination may be addressed by linking decisions within the same function at different echelons in the organization. A large vertically integrated firm may have a hierarchy of production plants making semi finished products for assembly into final products. Production decisions at these plants must be coordinated if the firm is to achieve the performance measure targets it has set for itself. In order to be effective, such coordination must take into consideration, the effects of, uncertainty of final demand, uncertainties in production process at each plant and capacity constraints at each plant. We refer to this second level of coordination as 'multi-plant coordination'. Each plant here refers to a manufacturing facility that is centered around related production processes. This paper focuses on issues that must be addressed for effective multi-

plant coordination. There is considerable overlap and interaction between the areas of general coordination and multi-plant coordination as defined above. However, there is currently no well defined framework in literature that explains such interaction. Moreover, there is no unified body of literature that deals comprehensively with either type of coordination.

The objective of this paper is to review related literature and to identify the important issues so that a coherent framework for multi-plant coordination can be proposed. This problem arises in vertically integrated firms. The objective of this problem is to coordinate the production plans of several manufacturing plants so that the overall performance of the firm is improved. This objective is eminently desirable. It enables firms to integrate the production planning function for the entire organization making plants more responsive to the needs of other plants. This would ensure that the firm as a whole benefits and improves its performance on various measures and thereby its competitive position.

The rest of this paper is organized as follows. In Section 2 we review literature pertaining to efforts in general coordination and develop a useful classification. In Section 3 we review efforts in multi-plant coordination. We conclude that any effort in multi-plant coordination must address the issues of nervousness of demand, lot sizing and provision for safety stock. In Sections 4, 5 and 6 we focus on the current state of research in the above areas and discuss implications for multi-plant coordination. Finally in Section 7 we enumerate some of the benefits of multi-plant coordination and suggest directions for future research.

2. General coordination

We refer to general coordination as an attempt by firms to integrate decisions pertaining to different functions e.g., production and distribution. Research efforts in this area have been directed towards coordinating the operations of firms with a multi-echelon production-distribution structure and measuring the effect of such coordination in terms of the impact on operational performance measures like total cost (setup and inventory holding), overall lead time, average service level, etc. We have classified general coordination research into three categories each representing attempts to coordinate different operations of the firm. These categories represent, respectively, integration of decision making pertaining to

- (i) supply and production planning;
- (ii) production and distribution planning;
- (iii) inventory and distribution planning.

Table 1 General coordination issues

| Supply and production planning | Production and distribution planning | Inventory and distribution planning |
|--------------------------------|--------------------------------------|-------------------------------------|
| Monahan (1984) | King and Love (1980) | Federgruen and |
| Rosenblatt and Lee (1985) | Williams (1981) | Zipkin (1984) |
| Lee and Rosenblatt (1986) | Blumenfeld et al. (1987) | Burns et al. (1985) |
| Banerjee (1986a,b) | Ishi, Takahashi and Muramatsu (1988) | Dror and Ball (1987) |
| Goyal (1988) | Chandra and Fisher (1992) | Chandra (1990) |
| Goyal and Gupta (1989) | | Anily and Federgruen (1990) |
| Zoller (1990) | | |



Table 1 presents a classification of the research in general coordination and later we describe the major issues in each category.

The importance of a coordinated relationship between the supplier and the buyer has been emphasized in literature. As Goyal and Gupta (1989) note, coordination between the supplier and buyer can be mutually beneficial to both. Studies on coordination between supplier and buyer have focused on determining the order quantity which is jointly optimal for both. Using such an order quantity may lead to increase in overall profits which could be shared in some equitable manner between the two parties. For a given annual supply, vendors are often interested in procuring large individual orders. Large orders are beneficial to suppliers because of potential savings in order processing costs, manufacturing setup costs and distribution costs. However a buyer would want to purchase his optimal order quantity each time since any deviation from this quantity would cause his total cost of inventory holding and ordering to go up.

Monahan (1984) proposed a quantity discount model in which he showed that the vendor could increase his profits by enticing the buyer to purchase a greater quantity in return for a discount on purchase price. He showed that by offering a discount to offset increased holding costs, the vendor could motivate the buyer to increase his order quantity by a factor $K = (S_2/S_1) + 1$, where S_2 is the vendor's fixed order processing and manufacturing setup cost and S_1 is the buyer's fixed order processing cost. The author uses a relatively strong assumption of lot for lot policy, i.e., the vendor's lot size (for production/ procurement) is identical to the order quantity of the buyer. Also the results suggest that since the buyer is compensated only exactly (and no more) for his increased holding cost (due to higher order quantity) he would be indifferent towards increasing his order quantity.

Bannerjee (1986a,b) generalized Monahan's model by incorporating the effect of the vendor's inventory cost which had been ignored by Monahan (1984) in calculating the discount that the vendor must offer to the buyer. Also see Joglekar (1988) for related work.

The assumption of identical lot size for vendor and buyer was relaxed by Rosenblatt and Lee (1985). They showed that the optimal ordering quantity of the vendor is an integer multiple of the buyer's order quantity and proposed a linear discount schedule to determine the optimal pricing policy which the seller could offer to the buyer. Given such a discount schedule, the authors showed that the buyer would optimize his total cost by revising his economic order quantity. This could be exploited by the supplier to increase his profits. This result is also shown to be stronger than that of Monahan (1984) because there is motivation for both the supplier and the buyer to increase the order quantity. The interested reader is referred to Goyal and Gupta (1989) for a comprehensive review of literature pertaining to buyer-vendor coordination.

It is evident from the above discussion that there are some limitations of the research on supplier-buyer coordination. First, most studies assume that the vendor faces a constant, deterministic demand. Second, the treatment of the production process at the vendor is a gross simplification of the actual situation, single machine, single product and incapacitated situations. Third, with the emphasis on just-in-time manufacturing, larger purchase quantities in order to get discounts would be difficult to justify as a matter of policy. However it must be noted that just-in-time purchasing can be successful only when demand is stable over time as noted by Karmarkar (1989). In situations where demand is dynamic (which is very often the case in real life) the research direction outlined by Goyal and Gupta (1989) is likely to be useful.

The second type of general coordination research treated in literature is at the level of integrating production planning and distribution planning. The decision issue that a production planner is concerned with is to determine optimal production/inventory levels for each product in every period so that the total cost of setup and inventory holding is minimized. On the other hand, the



distribution planner must determine schedules for distribution of products to customers so that the total transportation cost is minimized.

These two activities can function independently if there is a sufficiently large inventory buffer which completely decouples the two. However this would lead to increased holding costs and longer lead times of products through the supply chain. The pressure of reducing inventory and lead times in the supply chain has forced companies to explore the issue of closer coordination between production and distribution. Recently there have been some noteworthy attempts in this area. King and Love (1980) describe the implementation of a coordinated production-distribution system at Kelly Springfield, a major tire manufacturer with four factories and nine major distribution centers located throughout the United States. The authors present a case study describing a coordinated system for the manufacturing plants and distribution centers. The implementation of this system resulted in substantial improvements in overall lead times, customer service and average inventory levels. The annual costs were also reduced by almost \$8 million.

Williams (1981) considered the problem of joint scheduling of production and distribution in a complex network. The performance of a dynamic programming based algorithm was compared to several existing heuristics. The objective of the problem was to minimize average production and distribution cost per period. However the author assumes a constant demand rate and this limits the applicability of this work.

Blumenfeld et al. (1987) considered the problem of synchronized scheduling of production and distribution for a parts producer supplying parts to a final assembly manufacturer. The scenario that these authors considered assumed fixed transportation costs per shipment and one destination per part type.

Blumenfeld et al. (1987) reported the successful implementation of this research at the Delco electronics division of General Motors that resulted in a 26% reduction in logistics costs. Ishii, Takahashi and Muramatsu (1988) have described a model for minimizing the inventory of 'dead stock'. Dead stock refers to the amount of inventory which is left over at the end of the product life cycle. This issue is important in a case where the market for products changes rapidly. Cohen and Lee (1988) presented a model for coordinating decisions in a supply chain. They proposed a framework for the evaluation of a supply chain in terms of performance attributes like cost, manufacturing lead time, etc. Their analytical model seeks to address the following issues:

1. How can production and distribution control policies be coordinated to achieve synergies in performance?
2. How do service level requirements for material input, work-in-process and finished goods availability affect costs, lead times and flexibility?

The authors subdivided the problem into four sub-models each corresponding to major physical activities in the supply chain viz., material control, production, finished goods stockpile and distribution. Each sub-model had a cost associated with it comprising setup cost (wherever applicable), holding cost and shortage cost. The objective of the problem was to minimize the total cost over all the sub-models. The sub-models were linked together by means of 'local' service targets which were the fill rates that had to be satisfied for each sub-model.



An overall optimization model which minimizes the cost over all the sub-models involves a constrained, nonlinear optimization problem and is intractable. Instead the authors suggest a hierarchical heuristic, which decomposes the problem into sub-problems corresponding to each of the sub-models described above. Each sub-problem was optimized separately in a given sequence. The output of a sub-model solution was used as the input data for other sub-problems. This methodology yields an upper bound on the objective of minimum overall cost. The research represents an innovative attempt at integrating several subsystems in a supply chain.

Cohen and Lee (1989) consider resource deployment decisions in a global manufacturing and distribution network. Their work addresses issues that are specifically relevant for firms that source material globally. The objective used is to maximize global after tax profits. The model considers variable and fixed costs for procurement, production, distribution, transportation as well as the tariffs, duties and transfer pricing. The authors have assumed a standard fixed transportation cost for transporting items from one place to another. When considering production-distribution systems for bulky items like petrochemicals or for items that require considerable transportation through a distribution network, a more detailed treatment of transportation cost is necessary. One approach is to consider detailed vehicle routing rather than use a fixed transportation cost.

The aspect of integration of vehicle routing and production planning has been analyzed recently by Chandra and Fisher (1992). The authors have developed an integrated model to coordinate production scheduling at a manufacturing plant with the distribution policies to serve a set of geographically disbursed customers. They consider a plant (with a finished goods stockpile) which supplies finished goods to a set of retailers located over a large geographical area. The following trade-offs have to be considered in order to coordinate production and distribution decisions: (i) Large batches to meet production's objective of few setups pushes up the inventory of finished goods at the warehouse; (ii) Consolidating loads of different items to reduce transportation costs requires additional setups or inventory requirements; (iii) Frequent shipments may result in higher transportation costs and increased setup costs although inventory levels may be reduced. The proposed model coordinates the capacitated lot sizing problem (at the manufacturing plant) and the vehicle routing problem (for minimum cost distribution of finished goods to customers). The authors report a reduction in total operating cost, for a range of problem parameters, of 3% to 20% compared to an 'uncoordinated' approach where production and distribution decisions are made independently. They also suggest that the benefits of coordination increase as the length of the planning horizon, the number of products & retail outlets, and vehicle capacity increases. It is also found beneficial to coordinate these functional activities where production capacity at the plant is less binding, and distribution costs increase relative to production costs.

A number of authors have also addressed the third type of problem in general coordination, i.e., coordinating inventory planning with distribution planning. This problem considers the scenario where a number of customers have to be supplied from one or more warehouse(s). The decision problem is one of deciding the replenishment policy at the warehouse and the distribution schedule for each customer so that the total cost of inventory and distribution is minimized. The trade-off is one of reduction in inventory costs versus an increase in the transportation costs. For example, shipping in smaller quantities and with higher frequency would reduce the inventory level at the warehouse but would entail a higher transportation cost. Federgruen and Zipkin (1984) consider a one warehouse, multiple retailer systems and a single planning period but allow for random demands at the retailers. The authors show that the coordinated model results in



substantial cost savings. Bell et al. (1983) developed a computerized multi-period coordinated inventory control/ distribution scheduling model. Dror and Ball (1981) and Chandra (1990) have reported heuristic solution methods for coordinated multi period models. Chandra (1990) considers the case where the customers face dynamic demand. The minimization problem is treated over a finite planning horizon of discrete time periods and heuristic solutions are provided. The author compares the results of a coordinated model (warehouse ordering policy and distribution schedules to retailers determined jointly) with a base case where the two decisions are taken independently. The results show that significant savings may be achieved with the coordinated model. This is primarily due to the fact that replenishment at the warehouse occur as close as possible to the transportation. The author allows for products to be shipped to customers before due date (but not later). The findings indicate that the coordinated policy results in cost savings even if the plant incurs the holding cost for goods that were shipped to the customer before due date. This direction of research is important in cases where transportation is a substantial part of the overall cost for the operations of firms. Burns et al. (1985) developed an infinite horizon coordinated model for the above problem. Anily and Federgruen (1990) modeled this problem in a scenario where one warehouse supplies several geographically dispersed customers. The authors assumed that the customers face a constant demand although the rate could vary from one customer to another. The model is specific to the scenario wherein all customers are divided into various 'regions'. Each time one of the customers in a given region receives a delivery, this delivery is made by a vehicle that visits all other outlets in the region as well. Heuristics are presented for computing the upper and lower bounds on the system wide costs and these are shown to be asymptotically tight as the number of customers increases. Since the proposed heuristics attempt to link two very difficult problems, the above models may be particularly useful in addressing the multi-plant coordination problem which would require a similar linkage. An important extension of the above research could be to study the impact of errors in demand forecast and the role of safety stock at different locations.

In this section we have briefly surveyed models where authors have analyzed the impact of functional coordination. Most of the above models consider multi-item, single stage manufacturing systems and then try to improve the system performance by coordinating different functions. However when the product being manufactured is complex, for example in the case of computers, telecommunication equipment, etc., the processing is often divided between a number of plants. Such an organization of the production process among different specialized plants was called 'focused factories' by Skinner (1974). In such cases it is important to consider the problem of coordinating the production plans of the different manufacturing plants. This is the problem of multi-plant coordination and is the focus of the next section.

3. Multi-plant coordination

As mentioned earlier, the multi-plant coordination problem seeks to link together the production plans of several manufacturing plants which are part of a vertically integrated firm, i.e output from one plant becomes an input into another plant. The objective of such coordination is to achieve near optimal results on performance measures like total cost, manufacturing lead time etc., for the entire organization. Coordination efforts must model the impact that production planning at one plant has on production planning at another plant. Such models must also take into consideration uncertainties associated with both the demand and the production processes. Published work in this area is scant and to our knowledge not many researchers have addressed the above problem. A possible approach to this problem can be seen in the work of Cohen and



Lee (1988). In their work the authors model a serial multi-stage, batch production process. A product is allowed to be processed on more than one line. For each batch of a product processed at a workstation, the authors approximate the total production lead time by the weighted sum of setup times, processing times, material delay times and the waiting times at the workstations. The workstation is treated as an M/G /1 queue and this enables an estimation of the waiting time at the workstation in the spirit of Karmarkar, Kekre and Kekre (1983) and Zipkin (1986). The authors also make the approximation that the departure process from one workstation to another is poisson. Their work considers a transfer batch size between workstations of one unit. This assumption will be violated when we consider transfer between two plants. Their work also does not consider any capacity limitations on the production line. Nevertheless this study is noteworthy for being the first to describe a comprehensive coordination model.

Beek, Bremer and Putten (1985) have addressed the issue of flexibility and design in multi-level assembly systems. Flexibility can be achieved by cutting down setup costs, which reduces the batch size, and the assembly lead-time. Design issues relate to physical structuring of the assembly network. An industrial application of the model at Philips Industries in Eindhoven is described. The model is useful in comparing different assembly structures for assembling complex products. The batch size calculation takes into consideration the inventory holding costs, setup costs and assembly lead times. This research direction is important as the model coordinates operations at several facilities with the objective of reducing lead time. The authors assume constant demand and future research is needed to extend the findings to other demand scenarios.

Kumar et al. (1990) have considered a variant of the above production planning problem in a supplier-buyer scenario with uncertain but bounded demand conditions. They assume a Supply contract wherein the quantity to be supplied in each period is specified in the contract (for N periods) in terms of an upper bound (U) and a lower bound (L). At the beginning of each period the buyer specifies the actual quantity he will purchase and this quantity is contractually obliged to be between U and L every period. In certain cases, describing the demand by only U and L rather than approximating a distribution for the demand amounts to (possibly) ignoring available information. However the authors justify this on the basis that it makes the model much more easy to solve since exact closed form solutions are known to exist for only very simple problems (single period) under the assumption of stationarity of demand.

They also assert that in certain industries with short product life cycles, it may be difficult to gather sufficient data to deduce the demand distribution with a high degree of confidence and the demand patterns may show nonstationarity. The value of available information is traded off against the ease of solving the problem. Given this type of requirement specification (in terms of upper and lower bounds) for each future period, the plant manager at the supplier's plant must decide how much to produce in each period so that his total costs are minimized. The model charges holding costs against positive inventories and penalty costs against backorders. A state variable whose value can range between 0 and 1 determines the actual demand for product i in period t . The decision variables are the production quantities of each product in every period. Each decision is evaluated at each set of realized demands determined by the state variable. The objective is to choose the minimum of the maximum costs of all decisions (Minmax problem). The problem considered is a multi-period, multi-product, single stage one with capacity constraints. This research is also dependent on the type of supply contract specified by the authors and therefore the scope of this research is limited to the industries (e.g., semiconductor industry) where such contracts are applicable.



The above discussion brings out one critical issue that needs to be addressed when we attempt to coordinate the operation of multiple plants, i.e., the question of lotsizing. Lotsizing in a multi-plant scenario is complicated by the fact that lotsize at one plant is dependent on the lotsize chosen at another plant. One alternative to the above problem is to completely isolate the plants from each other by means of intermediate inventory. However the increased cost of inventory and the increased lead time for products through the supply chain makes this a poor choice. Another approach could be to determine the production plan for the plants level by level or hierarchically. Beginning with the plant that supplies the finished goods warehouse, the production plan is prepared and this defines the requirements for the previous plant. This procedure is continued till the production plan for all plants is prepared. Such a procedure ignores the interaction between various plants and will yield sub-optimal production plans. A lotsizing model in a multi-plant scenario must correctly account for the interdependence between different plants. Another issue is the actual implementation of the lotsizing algorithms. Most lotsizing algorithms are implemented on a 'rolling horizon' basis. This method of resolving the model at the beginning of each time period causes disruption of previously planned production activities which is known as 'nervousness'. In order to be effective, multi-plant coordination must consider the impact of nervousness on cost and lead time. Also, consideration of stochastic demand would make it necessary for managers to define safety stock levels necessary to maintain required customer service levels. Effective multi-plant coordination must be able to integrate the issues of lotsizing, nervousness and safety stock into a coherent framework. Attempts to address the multi-plant coordination problem can draw on the existing research on the above questions. We now discuss the major research efforts on nervousness, lotsizing and safety stock and classify these efforts in order to identify important issues. This is important for integrating the relevant issues so that a framework of multi-plant coordination may be proposed.

4. Nervousness issues

An important issue that arises in coordinating the multi-plant structure is the impact of 'nervousness' of demand on total cost and lead-time. Nervousness arises due to two reasons. The first reason for nervousness is the 'horizon effect'. Schedules are developed on a rolling basis wherein a sequence of production decisions is determined by successive solution of the finite-

horizon, multi-period model. The decision for the current period is implemented and as the period elapses, demand for a new period is appended to the horizon and the model is resolved with the additional information. This may lead to changes in the production plan in a later period which disrupts the schedules made earlier. This may also lead to increase in total cost of operation. The second reason that gives rise to nervousness is the 'new forecast' effect. As new and more accurate data regarding requirements in future periods becomes available, it is incorporated in the model to get a new production plan. Nervousness can be disruptive for manufacturing systems. For example, if a schedule revision specifies a setup in a period where no production was planned or considerably alters the production quantity, this will disrupt plans concerning personnel scheduling and machine loading. In a multi-plant structure such disruption can propagate to all plants necessitating frequent revision of production plans. It is therefore important in a multi-plant scenario to choose a strategy that maintains a balance between the disruptive effect of nervousness and the need to respond to new and more accurate information. Research efforts in nervousness pertaining to both horizon effect and new forecast effect are outlined in Table 2.



Table 2 Nervousness issues

| Horizontal effect | New Forecast effect |
|---------------------------|------------------------------------|
| <i>Single-stage:</i> | |
| Baker (1977) | Carlson et al. (1979) |
| Baker and Peterson (1979) | Blackburn and Millen (1980) |
| Sridharan et al. (1987) | Carlson, Beckman and Kropp(1982) |
| | De Bodt and Van Wassenhove (1983) |
| | Kropp and Carlson (1984) Ho (1989) |
| | Sridharan and Berry (1990) |
| <i>Multi-stage:</i> | |
| Chand (1983) | |
| Blackburn et al. (1986) | |

Baker (1977) has studied the efficiency of optimizing a finite-horizon, multi-period model for a single stage production system and implementing those decisions on a rolling basis. Finite horizons are used in production planning because of the limited availability of future demand data and the uncertainty associated with these data. The author focuses on the finiteness of the future information. The motivating question in this study was to find how good were optimal, static decision models for the system when implemented on a rolling basis. Given reliable but limited demand data for future demand, the Uncapacitated Dynamic Lot Sizing (DLS) Model (Wagner and Whitin, 1959) was used to evaluate the cost of implementing rolling schedules. The length of the rolling forecast horizon was varied and the schedule was rolled successively over 48 time periods. The solution of the DLS model for the entire 48 periods (assuming that this much information is available at the outset) was treated as the benchmark or the 'optimal' solution for making comparisons. The results showed that rolling schedules achieved costs within 10% of optimality. The choice of the most appropriate length of forecast horizon was dependent on whether or not the demand pattern was seasonal. Without seasonality, the best forecast period was found to be the 'natural cycle'. The natural cycle is the replenishment interval for the EOQ model. However when demand showed a seasonal pattern, the use of multiples of seasonal cycle as forecast horizon was found to be effective. The results of this study imply that the number of periods used as forecast horizon is a crucial parameter if rolling schedules are to be utilized effectively. The choice of a proper length of forecast horizon may be dependent on the demand pattern (whether seasonal or not). Once a good forecast horizon is chosen, the use of rolling horizons in a multi-period dynamic demand model can lead to efficient performance. However the systems considered are very simple uncapacitated, single stage with no forecast errors.

Baker (1977) also does not consider any specific cost of disruption of schedules. Disruption due to nervousness is clearly undesirable in production systems and has an associated cost. At the same time it may be economically infeasible to ensure perfect stability of plans. In this context it becomes important to define a reasonable level of nervousness. Carlson et al. (1979) address the issue of determining the amount of nervousness that can be deemed 'economically tolerable' in a manufacturing system. They suggest that practising managers have a tendency to tolerate non-optimality more than unstable schedules. However it may be a better strategy to strike a balance between the cost of dealing with nervousness (i.e., cost of schedule changes) and the cost of a



non-optimal solution (resulting from ensuring stability of plans). Nervousness imposes two kinds of costs:

- (i) cost of lot size changes for periods in which setups are already scheduled;
 - (ii) cost of scheduling setups in periods in which they were not previously scheduled ('new setups').
- The authors analyze the effects of only new setups and assume that the changes involved in the first category are far less costly to implement. This view is similar to the suggestions made by Mather 1977). The cost of scheduling a new setup depends critically on the period for which it is scheduled. New setups for the first several periods in the horizon may be impossible to effect due to unbreakable commitments and can be considered to have an infinite cost. On the other hand a new setup near the end of a long scheduling horizon may have a relatively low cost. The authors express the cost function of implementing schedule change in period $k, (V_k)$ as follows:

$$v_k = \begin{cases} \infty & \text{when } k = 1, 2, \dots, p, \\ f(k) & \text{when } k = p + 1, p + 2, \dots, r, \\ 0 & \text{when } k = r + 1, r + 2, \dots, N, \end{cases}$$

Where N is the length of the forecast horizon. In the first p periods, no schedule changes are allowed. The authors suggest that a reasonable value for p may be the minimum achievable leadtime offset for the item being currently produced. This implies that items have already been "released for final assembly and no changes can be made in the schedule. Similarly r might be set equal to the cumulative leadtime of all components and raw materials required to produce the item in question. This implies that the product has not entered even the first stage of manufacture and so changes can be made at very low costs. The objective of the model is to minimize the total cost function which consists of setup, holding and schedule change costs:

$$\begin{aligned} \text{Min } C &= \sum_{k=1}^N h_k I_{k+1} + \sum_{k=1}^N s_k \delta(x_k) \\ &+ \sum_{k=1}^N v_k \delta(\delta(x_k) - \delta(\hat{x}_k)) \\ \text{s.t. } I_{k+1} &= \sum_{j=1}^k x_j - \sum_{j=1}^k d_j, \end{aligned}$$

where

$$\delta(z) = \begin{cases} 1 & \text{if } z > 0, \\ 0 & \text{if } z \leq 0. \end{cases}$$

\hat{x}_k = Production Lot Size in period k in the existing schedule.

x_k = Production Lot in new schedule (to be determined).

I_k = Beginning inventory in period k .

d_k = Amount demanded in period k .



h_k = Holding cost per unit of inventory carried into period $k + 1$.

S_k = Setup cost.

V_k = Schedule change cost.

The algorithm is applied to analyze both the horizon effect and the new forecast effect for small problems (6 periods). The authors also perform sensitivity analysis to test the changes in the value of the optimal solution as a result of changes in the values of schedule change costs. This is important for the decision maker because he can be confident that the most imminent scheduling decision is optimal for a range of schedule change costs.

The methodology outlined in the last paper allows the manager to strike a balance between the cost of making schedule changes and the savings that such changes bring about. The result is neither complete dependence on repeated use of the scheduling algorithm whenever new information is obtained nor a striving for stability at all costs. Determining realistic values of p and r as well as the schedule change cost function $f(k)$ is important from the point of view of multi-plant coordination. The important issue here, not considered by the authors, is the relationship of these parameters to capacity. Disruption of planned production needs to be analyzed in terms of the additional demands it makes on the available capacity and the cost of providing additional capacity. Extreme instability of schedules can then be avoided by either freezing the production schedule over a period of time or by limiting the change in schedule to a specified limit. This is common in companies like Toyota (Monden, 1983), which have successfully implemented just-in-time production systems.

The approach suggested above is used by Sridharan et al. (1987) who analyze the effect of freezing a part of the master production schedule (MPS) in order to ensure stability in operations within the context of an MRP system. The authors consider uncapacitated lot sizing decisions using the Wagner-Whitin algorithm. Uncertainty in demand forecast is assumed to be negligible and safety stock is set to zero. Hence only nervousness due to horizon effect is considered. The objective of this research is to examine the impact of three important design factors in terms of cost and the stability of the MPS when lotsizing decisions are implemented on a rolling basis. Cost is represented by the percentage increase in total cost over the optimal cost. The schedule instability represents the average change in quantity per order over the simulation run and incorporates changes in both quantity and timing of the MPS orders. The factors considered are:

- (i) method used to freeze the MPS;
- (ii) proportion of the MPS that is frozen; and
- (iii) length of the planning horizon for the MPS.

Two methods can be used to freeze the MPS -specifying the number of periods (for which the MPS is frozen e.g., 1, 2, ..., N periods in future) or specifying the number of future orders (as determined in the current period) to be frozen. In the first case the schedule rolls over to the next period and the model is resolved. In the second case the schedule rolls over to the period immediately succeeding the last frozen order. The second factor i.e., proportion of the planning horizon to be frozen, is the ratio of the 'freeze interval' (number of periods frozen, K) and the length of the planning horizon (number of periods forming the planning horizon, N). The third factor is the length of the planning horizon (N) and this was expressed as a multiple (K) of the natural cycle (T). The analysis of results shows that cost error becomes significant only when the frozen portion of the planning horizon exceeds 50% of the total planning horizon and that the cost of freezing increases rapidly if more than 80% planning horizon is frozen. The cost error for freezing schedules for the period based model exceeds that for the order based procedure when the proportion of planning horizon frozen is greater than 0.5. This research demonstrates the



important effect of freezing the MPS on both the cost performance as well as schedule stability. However the assumption of certainty of demand limits the generalizability of the findings. The authors consider very simple single stage, uncapacitated systems.

The above work has been extended to the stochastic demand case in Sridharan and Berry (1990). Given stochastic demand and a required service level, the authors seek to determine the impact of design parameters for MPS freezing on cost and schedule instability under rolling planning. Cost and schedule instability are used in the same context as before. The design parameters considered are:

- i. the MPS lotsizing method;
- ii. planning horizon length;
- iii. frequency of planning production schedule;
- iv. proportion MPS that is frozen; and
- v. type of planning information used to freeze the MPS (based number of periods or orders).

Freezing a portion of the MPS provides a means of stabilizing plant and vendor schedules against nervousness due demand uncertainty. However freezing the MPS introduces a lag in responding to the changes induced by the uncertainty and this may lead shortages or excessive inventories at the MPS level. The results indicate that using order bas MPS freezing leads to more beneficial results against period based MPS freezing. Longer free intervals produce higher cost errors and reduce schedule instability using either type of freezing The cost errors become larger as the amount demand variability is increased. An interesting result is that in order to meet a given customer service level goal, an increased level of safety stock is required under frequent re-planning, producing an increase in the total production a inventory costs and hence the cost error. Another finding pertains to the length of the planning horizon. Longer planning horizons lead to increased cost error and this effect becomes more pronounced as the variability increases. These findings suggest that although the lead time considered may warrant the use of long term planning horizons, a reduction in the planning horizon length can lead to more stable schedules and a lower MPS lotsizing cost error when demand uncertainty exists. The results of the study provide important comparison of MPS freezing techniques under deterministic and stochastic demand conditions. However the study pertains t simplified single stage, uncapacitated case. An important direction of the future research could be to extend the above results to more complexing conditions.

De Bodt and Van Wassenhove (1983) the impact of forecast errors on total system (setup plus holding) under conditions of uncertainty. The authors consider the effectiveness of a single level lotsizing techniques in a rolling schedule environment with forecast errors. The performance of two well known heuristics Silver Meal Heuristic (SM) and Least Unit Cost Heuristic (LUC)) is analyzed under a constant demand pattern with normally distributed random errors. The presence of forecast errors leads to more frequent ordering as compared to the case when the demand is known with certainty. The authors show that the cost increase due to forecast errors results in an additional cost of continuously carrying half a period's demand. The major drawback of this study seems to be in its restrictive consideration of single level, uncapacitated systems, which limits its applicability. The results are also dependent on the assumption of level demand with normally distributed errors. Nevertheless it is among the first efforts to consider the effect of uncertainty in demand on lot sizing process within the MRP context

Blackburn et al. (1986) examine the effective- ness of alternative strategies for dealing with the problem of nervousness. The authors compare the relative effect of alternative strategies like freezing the master production schedule, use of safety stock, lot for lot policy, forecasting of demand beyond the planning horizon, etc. The authors suggest an alternative strategy which is



based on the work of Carlson et al. (1979). Each time new information is available, the model is resolved after modifying the setup cost for each period, depending on whether the item is scheduled in that period or not. The objective is to encourage setups in periods where they are scheduled previously and vice versa. This approach ensures that the schedule will change only when the joint consideration of the setup, carrying and the schedule change costs indicates that it is beneficial to do so.

The above review of literature relating to nervousness issues in production systems underlines

Table 3

Lotsizing issues

| Single stage | | Multi-Stage | |
|---|---|--|--------------------------------------|
| Incapacitated | Capacitated | Incapacitated | Capacitated |
| A. Single item | | | |
| <i>Exact approaches:</i> | | | |
| Wagner and Whitin (1948) | Florian and Klein (1971) Jagannathan and Rao (1973) | Zangwill (1969) Love (1972) Afentakis et al. (1984) | Lambrech and VanderEecken (1978a) |
| Zangwill (1969) | Love (1953) Swoveland (1975) Baker et al. (1978) Lambrech and VanderEecken (1987b) Chung and Ling (1988) | | |
| <i>Approximations :</i> | | | |
| De Matteis (1971) Silver and Meal (1973) | Bitran and Matsuo (1986) | Berry (1972) More (1974) New (1974) Coleman and Mcknew(1991) | |
| <u>B. Multi-item</u> | | | |
| <i>Exact Approaches :</i> | | | |
| | Barany et al. (1984) Van Roy and Wolsey (1987) Pochet and Wolsey (1991) | Zangwill (1966) Crowston and Wagner (1973) Crowston et al. (1973) Steinberg and Napier (1980) | |
| <i>Approximations :</i> | | | |
| | Manne (1958) Dzielinski and | Blackburn and Millen (1982) | Billington, McClain and |



| | |
|-------------------------|---------------------|
| Gomory (1965) | Thomas (1983) |
| Lasdon and | Zahorik, Thomas |
| Terjung (1971) | and Trigeiro (1984) |
| Lambrecht and | |
| VanderVeken (1979) | |
| Dixon and Silver (1981) | |
| Thizy and Van | |
| Wassenhove (1985) | |
| Karmarkar and | |
| Schrage (1985) | |
| Eppen and Martin (1987) | |
| Trigeiro et al. (1989) | |
| Lozano et al. (1991) | |

some of the basic trends. Current research has focused on planning horizon length as the production schedule and its cost has been found to be sensitive to the planning horizon considered. The cost of disruption due to schedule changes has also been addressed although these costs are hard to establish. Since production planning is implemented in multi-plant firms on a rolling basis, nervousness issues need to be addressed in such a scenario. Most authors have focused on single stage, uncapacitated systems. This is unrealistic in real life situations. More realistic systems need to be considered in future research where the propagation of nervousness through several plants can be correctly accounted for. There is also a need to study nervousness in terms of capacity requirements as well as the cost of providing additional capacity. For discrete parts manufacturing firms, setup time needs to be used in the models because in these firms there is no setup cost for items in terms of actual cash out-flow but the only cost is in terms of the time consumed.

Another direction of research could be to consider bounds on forecast revision. As pointed by Kumar et al. (1990), the supplier-buyer relationship in the semi-conductor industry is coordinated by a contract which specifies the upper and lower bounds on demand. This mechanism is also used by firms in computer industry, operating in a multi-plant scenario. Given such bounds on forecast revision, algorithms need to be developed for guiding managers in the choice of lotsize, so that required customer service is achieved.

5. Lotsizing issues

The multi-plant structure is a complex multi-stage manufacturing system. Each plant itself represents a multi-stage system in which the flow of products may be serial, parallel, assembly or general (Billington et al., 1983). Lotsizing is important when the operations of multiple plants is considered under tight capacity constraints. The problem is complicated by the interdependence of different plants. Two distinct issues need to be addressed. First, each individual plant needs to be represented by a simpler but an equivalent system which captures the salient features of the original plant, especially capacity usage. Second, a suitable lotsizing technique needs to be developed which can be applied to the simplified system. These issues can draw on the considerable literature on lot sizing. In this section we briefly review literature on single-stage and multi-stage lotsizing. We present a classification of some of the major problem types, which have been tackled till now, in Table 3.

The dynamic programming based solution procedure for the uncapacitated, single item dynamic demand situation, proposed by Wagner and Whitin (1958) has served as an important paradigm



for lot sizing analyses. Approximate solutions to the uncapacitated, single item, single stage model have been suggested by De Matteis (1971) and Silver and Meal (1973). The major advantage of these approaches is that they are computationally much more efficient than the exact solutions. Zangwill (1969) extended the basic model to include backlogging of demand. However none of the above models takes into consideration the finite processing capacity of the manufacturing facility. The inclusion of this constraint considerably complicates the analysis.

Florian and Klein (1971) devised a dynamic programming based shortest path algorithm for the case with constant capacity in every period with concave production and storage costs. The authors showed that the optimal solution to the above problem consists of independent sub-plans wherein the inventory level is nonzero in every period except the last. In the sub-plans the production level, if positive, is at capacity except for at most one period. Love (1973) developed an optimal schedule for the concave cost model with constraints on production and inventory in each period. Using network flow concepts the author showed that for arbitrary bounds on production and inventory there is an optimal schedule such that if for any two periods production does not equal zero or its upper or lower bound; then the inventory level in some intermediate period equals zero or its lower or upper bound. An algorithm for searching for such schedules is provided.

Swoveland (1975) developed a shortest path procedure for this problem with piecewise concave production and holding costs. Jagannathan and Rao (1973) consider the above production planning problem for a generalized cost function with bounds on backlogging, inventory and production capacity. Baker et al. (1978) present a tree search capacity constrained dynamic demand problem.

Lambrecht and VanderEecken (1978b) also present a model for the capacity constrained lot-sizing problem with different production cost and holding/ shortage cost functions than those used by Baker et al. (1978). Barany et al. (1984) solved the multi-item capacitated lotsizing problem to optimality by adding strong valid inequalities. Also see Van Roy and Wolsey (1987) and Pochet and Wolsey (1991) for related work.

Most of the above models for capacitated, single stage lotsizing models have been tested only on small to medium sized problems and analysis of these algorithms indicates that efficient heuristics would be necessary for larger problems, since running times would increase substantially for the latter. Bitran and Matsuo (1986) have studied approximate formulations for the above problem. They proposed heuristics for two alternative forms of the above problem and showed that under mild conditions of forecast error these forms of the problem are equivalent to the original problem. The heuristic procedure proposed by the authors is shown to be pseudo polynomial.

For the multi-item case, algorithms for solving the capacitated lotsizing problem (CLSP) have been suggested by Lambrecht and VanderVeken (1979), Dixon and Silver (1981), Thizy and Van Wassenhove (1985) and Maes and Van Wassenhove (1986a,b). These algorithms do not take setup time into consideration. Pioneering work on the CLSP with consideration of setup times was done by Manne (1958), Dzielinski and Gomory (1965) and Lasdon and Terjung (1971). These three papers are based on similar formulations of the above problem. Manne (1958) showed that the linear programming solution to the problem provides a good solution whenever the number of items is large compared to the number of periods in the planning horizon. Dzielinski and Gomory (1965) used Dantzig and Wolfe (1960) decomposition while Lasdon and Terjung (1971) used a generalized upper bounding procedure to obtain efficient solutions to large



problems. As explained by Trigeiro, Thomas and McClain (1989), the CLSP with setup times becomes considerably difficult so much so that even proving the feasibility of the problem is NP complete.

The authors have proposed a Lagrangian relaxation based primal-dual heuristic approach to solve the CLSP with setup times. The heuristic consists of the primal, smoothing and the dual procedures. In the primal procedure, the resource capacity constraints are relaxed and the Lagrangian problem (single item multi-period scheduling problem) is solved using Wagner-Whitin algorithm. The smoothing procedure is then applied by shifting and splitting scheduled lots as necessary to eliminate overtime. In the dual procedure the Lagrange multipliers of the capacity constraints are updated using sub gradient optimization. The algorithm then loops back to the primal procedure with the new values of the Lagrange multipliers. The Lagrangian solutions generated in each iteration are lower bounds to the optimal solutions while the solutions obtained from the smoothing procedure are the upper bounds. The approach suggested by the authors is particularly relevant to discrete manufacturing systems since the setups in such systems do not involve a significant cash outflow. Instead the setup cost is reflected as the opportunity cost of the time that is spent on setup. This cost becomes especially important in systems, which have tight capacity constraints. See Lozano et al. (1991) for related work.

Models for multi-stage systems have been proposed both within the MRP framework and as general production models. Zangwill (1969) and Love (1972) have developed efficient dynamic programming based algorithms for uncapacitated serial systems. Love (1972) shows that the optimal policy is nested for concave production and storage costs if storage costs are nondecreasing in order of facility and production costs are non-increasing in time. Nested policy implies that if a facility orders an item in a particular period, all downstream facilities also order in that period. Crowston et al. (1973) consider the issue of lot sizing in multi-stage assembly systems. The authors propose a dynamic programming algorithm for such a system with constant, continuous final product demand and infinite planning horizon. They show that under the assumption of time invariant lot sizes, the optimal lot size at each facility is an integer multiple of the lot size at the successor facility. Steinberg and Napier (1980) develop an optimal procedure for the multi-period, multi-product, multi-level lot sizing problem by modeling the system as a constrained generalized network problem with fixed charge arcs and side constraints. The resulting minimum cost flow problem yields optimal lot sizing decisions at all levels. Blackburn and Millen (1982) proposed a heuristic procedure to plan lot sizes in multi-stage assembly structures. Independent demand exists for only the final product and is assumed to be deterministic but dynamic. All lead times are assumed to be constant and no backlogging is allowed. Further, no capacity constraints are considered at any stage. The authors apply sequentially a single stage algorithm with a set of modified costs to account for the interdependencies among stages. The intention here is similar to transfer price mechanism, i.e., to achieve coordination of decisions at different stages of the process without a central planning unit dictating production schedules. The results of the heuristic are compared to optimal results obtained by solving the multi-stage lot sizing problem to optimality. The results show that the deviation from optimal results increases as the number of levels in the product structure increases. The advantage of this approach is that it retains the simplicity of single stage lot sizing algorithms and gives reasonably accurate results for product structures that do not have many levels. Capacity constrained extensions of the above model for general product structures are likely to be useful, given the intractability of optimal algorithms for multi-stage lot sizing. Billington et al. (1983) have introduced the idea of 'product structure compression' which has the objective of reducing the size of the problem while retaining the salient features of the problem in



terms of demand, cost, lead times and capacity requirements. The authors suggest that in most production systems there are only a few 'constrained facilities', i.e., work centers where capacity is likely to be a binding constraint so as to cause scheduling difficulty. Lotsizing is critical only for the constrained work centers and other work centers can often be scheduled on a lot for lot basis. Karmarkar et al. (1992) concur with this view when they talk about an approximate composite model' to represent a manufacturing system.

The above discussion on lotsizing touches on one area in this multi-issue domain. From the point of view of multi-plant coordination, there are two promising directions for research. The first area to focus on is to work on approximate representations of single plants in the spirit of Billington et al. (1983) such that the number of work centers representing a plant are reduced considerably. This representation must be able to capture the salient features of the original system. The objective here is to reduce a complex manufacturing system into its most critical 'constrained facilities'. Once this is achieved, it may be easier to use this approximate representation of a plant in a multi-stage lotsizing algorithm where each stage is an individual plant. The second area is the development of robust heuristics which capture the interaction between the plants. Consideration of setup times in the spirit Trigeiro, McClain and Thomas (1989) is an important criterion in discrete parts manufacturing systems. Clearly this is a very difficult problem to solve optimally. However good heuristics would help quantify the benefits of coordination as compared to the current practice of optimizing the objective plant by plant which ignores the interlinkages between plants. One more direction needs to be investigated in this context. Usually, firms establish operational performance measures at a higher strategic level of decision making. These measures take into consideration, the firm's priorities, competitive environment and industry norms. A two level procedure may be envisaged in such a situation where operational performance measures targets have been defined. First an optimization based heuristic is solved to determine lotsizes and these are plugged into a detailed simulation to check whether the targets are achieved.

6. Safety stock issues

In the previous section we discussed models for lotsizing in multi-stage production systems. Most of these models assume deterministic demand. However this is rarely true in real situations. The use of safety stock is widely prevalent in industry to counter variability that may be present in various forms, e.g., variability in demand forecast, variability in processing time or yield, variability in vendor replenishment time and quantity, etc. Products also have to compete for limited processing time and resources at each stage.

Consequently the manufacturing system does not have full flexibility to reschedule to combat the above forms of variability. Setting of safety stock in a multi-plant scenario is further complicated on account of the inter-dependencies which exist between plants. An important issue that arises in this context is the determination of safety stock at each plant if the firm has to achieve a pre-specified customer service level. As we pointed out in our discussion of nervousness, firms with a multi-plant structure often consider bounds on forecast revision for each plant, to limit schedule nervousness. This may be treated as a limit on the flexibility of the plants to adjust their level of production to changing forecasts. If however the firm wishes to provide a better service level, use of safety stock may be necessary. In this section we briefly review research on safety stock from the point of view of multi-plant coordination. The interested reader is referred to Graves (1988) for a more complete review of safety stock in manufacturing systems. A classification of the work on safety stock is presented in Table 4.

Clark and Scarf (1960) presented an optimal inventory policy for a serial system with stochastic



demand. The authors assumed a linear processing cost and a linear inventory holding cost. No ordering costs are considered. The objective used was to minimize the expected discounted costs. The optimal policy is computed by solving a series of one-stage inventory problems. Beginning with the last stage the optimal policy is computed under the assumption that sufficient input is available from the previous stage. From this optimal policy for the last stage, the authors then determine the costs imputed on the downstream stage by a stockout at the upstream stage. This analysis is successively carried on to the upstream stages. The model has been extended to the case of an assembly system (two components with differing lead times being assembled into a single end item) by Schmidt and Nahmias (1985). However this modest change in structure makes the analysis considerably difficult. It would therefore appear that extension of the above models to general structures would be difficult. Given the complexity of exact analysis, the focus should largely be on good heuristics.

Approximation models for setting safety stock in manufacturing systems fall into two categories. The first category is where lotsizing is not considered, i.e., lot for lot policy is followed with each stage ordering in every period. For this lotsizing policy, Simpson (1958) argues that planning must be done for a maximum reasonable demand which has been pre-specified. Each stage must be able to always fulfill the request of the downstream stage under such demand conditions and safety stock must be planned likewise. The inherent assumption here is that when an extraordinary demand situation arises, the system will take extraordinary actions (like expediting, etc.). Hence the manager only needs to plan safety stock for satisfying the maximum reasonable demand. This idea is in consonance with the idea of a bound on forecast revision that came up in our discussion on nervousness. The authors show that the optimal policy is an 'all or nothing' policy, i.e., either there is no inventory between two stages or there is sufficient inventory to completely decouple the two stages.

Hansmann (1959) considers a similar problem except that he assumes that there can be a delay in supplying the demand of the downstream stage. The poorer the service provided by the upstream stage, the longer will be the replenishment lead time for the downstream stage and more excess inventory will be needed. In this case the optimal policy turns out not to be an 'all or nothing' policy. Miller (1979) introduced the concept of hedging which consists of inflating the master production schedule to reflect the uncertainty in the end item demand.

Table 4
Safety stock issues

| Type of analysis | Uncapacitated | Capacitated |
|--|--|-------------------------|
| Exact analysis | Clark and Scarf (1960) Schmidt and Nahmias (1985) | |
| Approximations without lotsizing (i.e., lot for lot) | Simpson (1958) Hansmann (1959) Miller (1979) Wijngaard and Wortmann (1985) Graves (1988) | |
| Approximations with lotsizing | Clark and Scarf (1962) Lambrecht et al. (1984) Carlson and Yano (1986) | Lambrecht et al. (1985) |



The second category of approximation methods for setting safety stocks consists of those models that consider lotsizing. Clark and Scarf (1962) extend their previous work to allow for a fixed ordering cost at each stage. Their model computes an (s, S) policy for each stage with successive stages being linked by a penalty cost of a stockout. Lambrecht et al. (1984, 1985) extend the above analysis to assembly systems and to the case where downstream lot sizes are greater than the upstream lot sizes. The authors suggest that protection against uncertainty may take the form of either safety stocks or safety time. Safety time is the time between the production batch or procurement lot becoming available and the time when it is needed to produce some subsequent assembly or finished product. The authors use a Markov decision process based analysis to provide insight into near optimal policies for the above systems.

Carlson and Yano (1986) address the issue of determining cost effective safety stock levels for each item in the product structure under stochastic demand. A single product is considered which is assumed to have a multi-level assembly structure. The objective is to minimize average total setup and holding costs per period subject to achieving a specified customer service level. The problem is a general nonlinear (nonconvex) stochastic integer optimization problem and is computationally intractable. The authors used a heuristic approach and proposed upper and lower bounds on the optimal solution. The results indicate that safety stocks can be utilized beneficially at production stages where setup and disruption costs are high. This has important implications for the multi-plant scenario where we need to determine the amount of safety stock that must be carried at each plant. However, this study ignores capacity constraints. Consideration of capacitated systems is especially relevant for multiplant coordination. Research needs to be done to establish how capacities of different plants affect the need and level of safety stock for achieving a given customer service level.

Graves (1988) addresses the issue of planned lead-time for each stage, which serves as the target figure for this stage. The author suggests that the greater is the planned lead-time, smoother is the aggregate output and hence lower is the required production system flexibility. However with higher planned lead-time, the inter-stage and intra-stage inventory also increases. Therefore the trade-off examined is between inventory cost and production flexibility. Should we eliminate the need for production system flexibility by having long lead times and as a consequence carry greater amount of work in process inventory? Or alternatively should the production system be designed to be flexible in terms of varying load from period to period? The current emphasis on reduced inventory and lead-time clearly makes the first alternative unattractive to implement. The issue of capacity is once again ignored. Research needs to be done to link planned lead times, setup times and capacity of equipment.

The above discussion brings up some of the important issues that need to be addressed for multi-plant coordination. First there is need for establishing the concept of a maximum reasonable demand. This is the maximum level of demand for which safety stock planning needs to be done. This concept needs to be integrated with the idea of limited production flexibility as used by Graves (1988), i.e., the limited ability of a plant to adjust its capacity to changing forecasts. This limit could conform to the forecast revision bounds that have been discussed in the section on nervousness. This limit on forecast bound represents the flexibility of the system to adjust its output to match changed forecasts. Safety stock needs to be planned only for demand beyond this bound. Research is needed to clarify the above issues. There is also a need to establish interlinkage between plant capacity and the need for safety stock.



7. Conclusions

In this paper important issues that concern the multi-plant coordination problem have been analysed. This problem arises in vertically integrated firms. The objective of this problem is to coordinate the production plans of several manufacturing plants so that the overall performance and the competitive position of the firm is improved.

At a higher level of decision-making, the problem of general coordination needs to be addressed. This problem seeks to coordinate different functions like supply planning, production and inventory planning, distribution planning, etc., and is important for the multi-plant coordination problem because of the overlap between the two problems. The two problems are also similar in structure. Research efforts are required to establish an overall framework, which links these two problems.

For a framework for capturing the benefits of multi-plant coordination, the critical issues of nervousness, lotsizing and safety stock need to be integrated as these have a considerable impact on a firm's performance. Nervousness of schedules arises as new demand forecasts become available and are used to revise earlier production plans. The propagation of nervousness is important in a multi-plant scenario and has not been addressed in literature. Research is also needed to establish linkage between capacity and nervousness. Excessive nervousness is controlled in practice by either freezing production schedule for some length of time or by specifying limits on forecast revision. An important area of research is to establish how forecast revisions within bounds can be integrated with the lotsizing exercise. The issue of lotsizing needs to be addressed for capacitated multi-plant systems with explicit consideration of setup times although this is known to be a very difficult problem. This is especially important for discrete parts manufacturing firms as this correctly reflects the true situation in such firms. Given the intractability of the exact approach, heuristics are required so that good bounds can be established on the optimal solution. Individual plants need to be approximated in terms of the critical 'constrained facilities', i.e., facilities for which the capacity constraint is a binding one, such that the approximation captures the real situation in the actual plant.

A hierarchical approach may be necessary for solving the complete problem. First the multi-plant problem may be solved by considering only the constrained facilities at each plant. Once a good solution is available the detailed situation at each plant can be simulated and an interface developed with the higher-level model.

Finally the treatment of safety stocks is also important for multi-plant coordination. Safety stock and production flexibility are the two mechanisms which managers use for dealing with up-dated forecasts in multi-plant firms. Production flexibility is the ability of the system to alter the level of output to match changed forecasts. Bounds need to be established on the level of demand forecasts that each of the above mechanisms has to respond to. The first bound relates to the production flexibility of the system. Within this bound, the firm's production manager would be expected to adjust the output to match changed forecasts. This would be based on the amount of nervousness, which is considered tolerable in the system. A second bound on forecast revision would be needed for planning for safety stocks. This represents the maximum reasonable demand and safety stocks would be planned for demand between the first and second bounds. Further research needs to be done to clarify the above framework for multiplant coordination.



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